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FOR XII

MATHEMATICS
(Subject Code-041)

O.P. GUPTA
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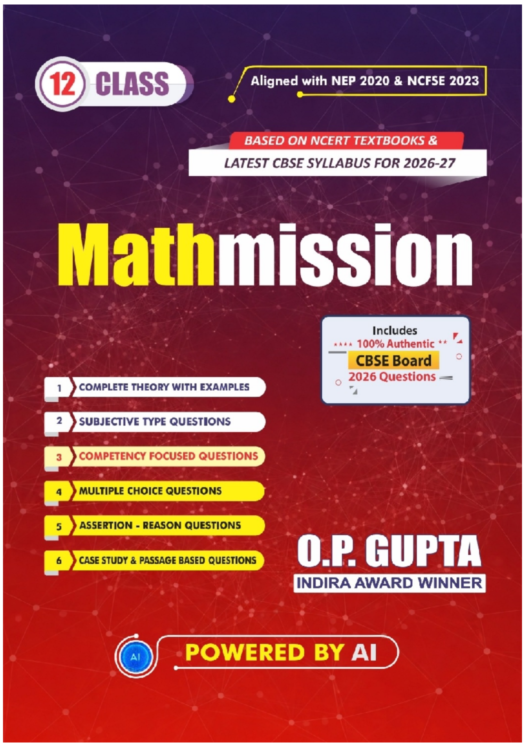

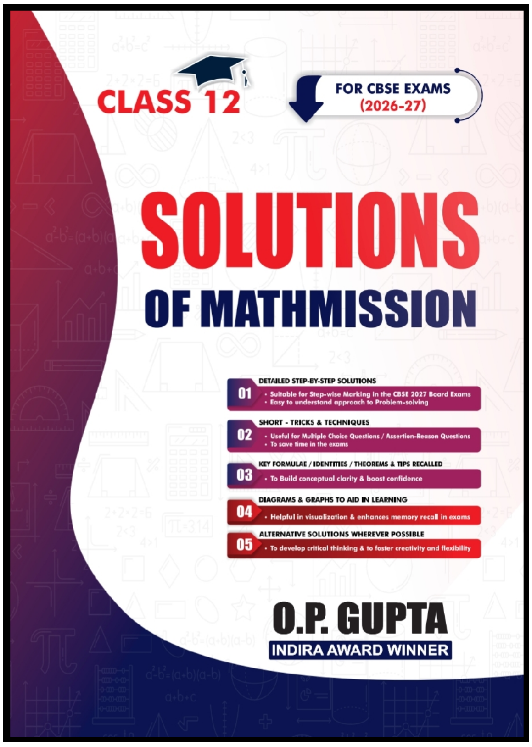

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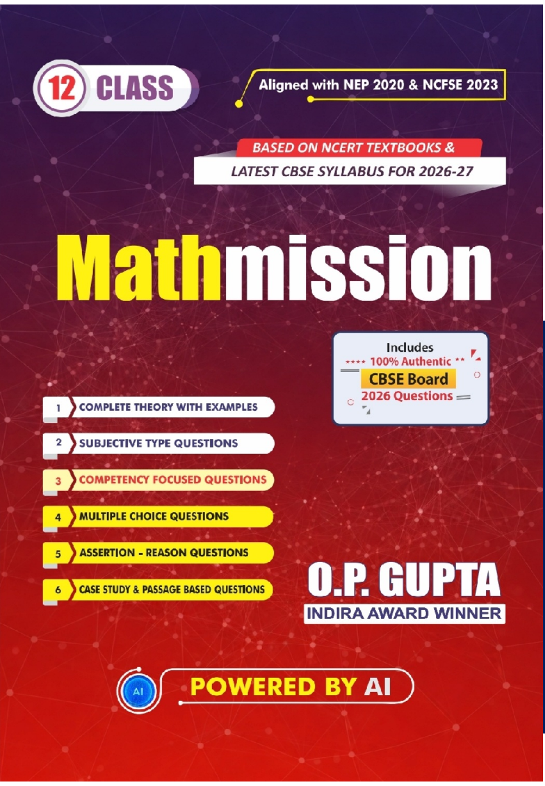

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MATHMISSION FOR XII

For CBSE 2026-27 Exams ▪ Class 12 Maths (041)

By O.P. Gupta

Indira Award Winner

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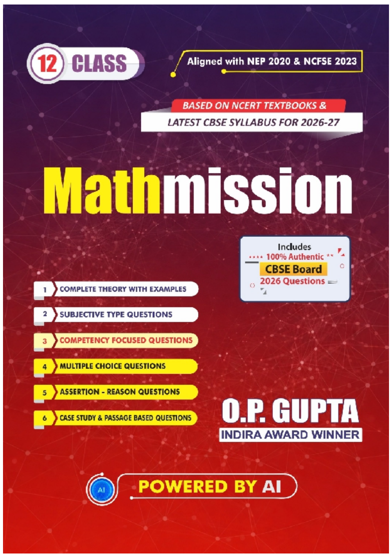

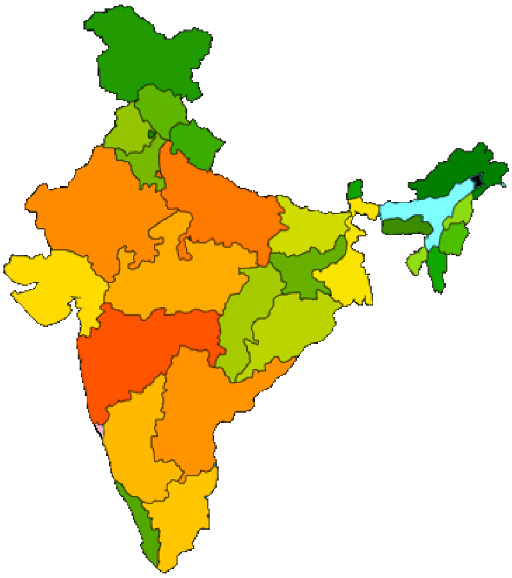

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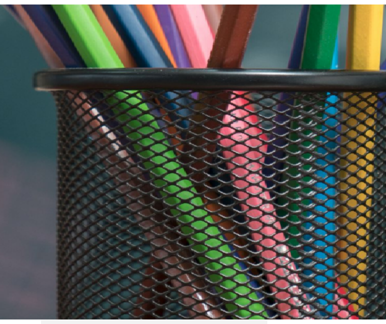
TABLE OF CONTENTS



CHAPTER 01

ALGEBRA OF MATRICES & DETERMINANTS

All mathematical truths are relative and conditional.

In this chapter, we shall learn

- ✓ Definition of Matrix, notation, related terminologies & type of matrices
- ✓ Algebraic operations on matrices viz. addition, subtraction and multiplication
- ✓ Understanding various properties viz. commutative, associative properties for algebraic operations on matrices, Equality of matrices
- ✓ Transpose of matrix, Symmetric and skew-symmetric matrices
- ✓ Existence of inverse of a matrix; problems based on the definition of inverse of matrices
- ✓ Defining determinant of a square matrix (up to 3^{rd} order matrices)
- ✓ Minors, Co-factors, Application of determinants in finding the area of Δ
- ✓ Properties of determinants
- ✓ Adjoint of matrix and inverse of square matrix by determinant method
- ✓ Consistency & inconsistency of system of linear equations (two or three variable system of linear equations) and their solutions using inverse of matrix
- ✓ Real life - Application based Problems

✪ BASIC ALGEBRA OF MATRICES

INTRODUCTION

Matrices are very powerful tools not only in the field of Maths but also in Economics, Computers, and Cryptography etc. In computer based programming, these matrices play a vital role in the projection of three-dimensional image into a two-dimensional screen, creating the realistic motion pictures. Matrices and their inverse matrices are used by a programmer for coding or encrypting a message. A message consists of a sequence of numbers in a binary format that is used for the communication. The process of coding and decoding requires coding theory that involves solving the linear equations. These equations are solved with the help of matrices. With these encryptions only, the internet is functioning and even financial institutions are able to transmit sensitive and private data securely.

IMPORTANT TERMS, DEFINITIONS & RESULTS

01. Matrix - a basic introduction :

Def. A matrix is an ordered rectangular array of numbers (real or complex) or functions which are known as *elements* or the *entries* of the matrix.

A matrix is denoted by the **upper case letters** *i.e.* A, B, C etc.

The array is enclosed by brackets $[\quad]$, the parentheses $(\quad)$ and the double vertical bars $\|\quad\|$.

For example, $B=\begin{bmatrix}1 & \sqrt{2}\\ 0 & 6\end{bmatrix}$, $M=\begin{bmatrix}4 & 7 & 0\\ -1 & 5 & 9\end{bmatrix}$, $C=\begin{bmatrix}3\tan x-\sin x\\ \cos x+2\sec x\end{bmatrix}$, $D=\begin{pmatrix}1 & x & 2-x\\ x^2 & 4 & 6\\ x+3 & 5x^2 & 0\end{pmatrix}$.

☑ Consider a matrix A given as, $A=\begin{bmatrix}a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n}\\ \vdots & \vdots & & \vdots & & \vdots\\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in}\\ \vdots & \vdots & & \vdots & & \vdots\\ a_{m1} & a_{m2} & \dots & a_{mj} & \dots & a_{mn}\end{bmatrix}_{m\times n}$.

Here in the matrix A depicted above, the horizontal lines of elements are said to constitute ***rows*** of the matrix A and vertical lines of elements are said to constitute ***columns*** of the matrix A .
Thus matrix A has ***m*** **rows** and ***n*** **columns**. Note that, $m, n \in \mathrm{N}$ (set of natural numbers).

- *A matrix having m rows and n columns is called a matrix of order $m \times n$ (read as '**m by n**' matrix). And a matrix A of order $m \times n$ is depicted as $A = \left[a_{ij}\right]_{m\times n}$; $i, j \in \mathrm{N}$.*
- *Also in general, a_{ij} means an element lying in the i^{th} row and j^{th} column.*
- *No. of elements in the matrix $A = \left[a_{ij}\right]_{m\times n}$ is given as $(m)(n)$.*

02. Types of Matrices :

a) Column matrix :

A matrix having only one column is called a *column matrix* or ***column vector***.

e.g. $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}_{3\times 1}$, $\begin{bmatrix} 8 \\ 5 \end{bmatrix}_{2\times 1}$.

✪ **General notation: $A = [a_{ij}]_{m\times 1}$.**

b) Row matrix :

A matrix having only one row is called a *row matrix* or ***row vector***.

e.g. $\begin{bmatrix} -1 & 2 & \sqrt{3} & 4 \end{bmatrix}_{1\times 4}$, $\begin{bmatrix} 2 & -5 & 0 \end{bmatrix}_{1\times 3}$

✪ **General notation: $A = [a_{ij}]_{1\times n}$.**

c) Square matrix :

It is a matrix in which the number of rows is equal to the number of columns i.e., an $m \times n$ matrix is said to constitute a square matrix if $m = n$ and is known as a ***square matrix of order 'n'***.

e.g. $\begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & -4 \\ 0 & -1 & -2 \end{bmatrix}_{3\times 3}$ is a square matrix of order 3; $\begin{pmatrix} \frac{3}{8} & -\frac{4}{7} \\ 5 & \frac{9}{11} \end{pmatrix}_{2\times 2}$ is a square matrix of order 2.

✪ **General notation: $A = [a_{ij}]_{n\times n}$.**

d) Diagonal matrix :

A square matrix $A = [a_{ij}]_{m\times m}$ of order 'm' is said to be a *diagonal matrix* if $a_{ij} = 0$, when $i \neq j$ *i.e.,* all its **non-diagonal** elements are *zero*.

e.g. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times 3}$ is a diagonal matrix of order 3; $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 8 \end{bmatrix}_{3\times 3}$ is a diagonal matrix of order 3.

Also $\begin{bmatrix} 1 & 0 \\ 0 & -5 \end{bmatrix}_{2\times 2}$ is a diagonal matrix of order 2.

- *Also there is **one more notation** specifically used for the **diagonal matrices**.*
For instance, consider the matrix depicted above, it can be also written as ***diag.$(2 \quad 5 \quad 4)$***.
- *Note that the elements $a_{11}, a_{22}, a_{33}, \ldots, a_{mm}$ of a square matrix $A = \left[a_{ij}\right]_{m\times n}$ of **order m** are said to constitute the **principal diagonal** or simply **the diagonal of the square matrix A**. These elements are known as **diagonal elements of matrix A**.*
- *The secondary diagonal of a square matrix is the diagonal that runs from the lower left corner to the upper right corner. It's also known as the minor-diagonal, counter-diagonal, or anti-diagonal.*

e) Scalar matrix :
A *diagonal matrix* $A=[a_{ij}]_{m\times m}$ is said to be a *scalar matrix* if its *diagonal elements* are **equal** *i.e.*,
$$a_{ij}=\begin{cases}0, \text{ when } i\neq j\\ k, \text{ when } i=j \text{ for some constant } k\end{cases}.$$
e.g. $\begin{bmatrix}5 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 5\end{bmatrix}_{3\times 3}$ is a scalar matrix of order 3. *Is it a diagonal matrix?*

*Can you define **scalar matrix** using a square matrix?*

f) Unit matrix or Identity matrix :
A square matrix $A=[a_{ij}]_{m\times m}$ is said to be *identity matrix* if the element a_{ij} is given by
$$a_{ij}=\begin{cases}1, \text{ if } i=j\\ 0, \text{ if } i\neq j\end{cases}.$$
A *unit matrix* can also be defined as the *scalar matrix* each of whose diagonal elements is *unity* (i.e., 1).
We denote the identity matrix of order m by I_m or, I.

e.g. $I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$, $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$.

Here first identity matrix is of order 3 whereas the second identity matrix is of order 2.
Do you agree that both of these matrices in above examples can also be treated as Scalar matrices?

g) Zero matrix or Null matrix :
A matrix is said to be a *null matrix* if each of its elements is '0' (zero).
It is denoted by English alphabet 'O'.

e.g. $\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$, $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$, $\begin{bmatrix}0 & 0\end{bmatrix}$.

h) Horizontal matrix :
A $m\times n$ matrix is said to be a *horizontal matrix* if $m<n$ i.e., if number of rows is less than the number of columns in the matrix.

e.g. $\begin{bmatrix}1 & 2 & 0\\ 5 & 4 & 7\end{bmatrix}_{2\times 3}$, $\begin{pmatrix}1 & 2 & 0 & \frac{5}{7}\\ -4 & 3 & 7 & 9\\ \sqrt{5} & 6 & \frac{1}{2} & 8\end{pmatrix}_{3\times 4}$.

i) Vertical matrix :
A $m\times n$ matrix is said to be a *vertical matrix* if $m>n$ i.e., if number of rows is more than the number of columns in the matrix.

e.g. $\begin{pmatrix}2 & 5\\ 0 & 7\\ 3 & 1\end{pmatrix}_{3\times 2}$, $\begin{bmatrix}4 & -5 & 7\\ 0 & 1 & \sqrt{3}\\ 5 & 6 & 9\\ 8 & -1 & 2\end{bmatrix}_{4\times 3}$.

j) Triangular matrices :

☑ **Lower triangular matrix**

A square matrix is called a lower triangular matrix if $a_{ij} = 0$ when $i < j$.

e.g. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 5 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 5 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 4 & 5 & 7 \end{bmatrix}$.

☑ **Upper triangular matrix**

A square matrix is called an upper triangular matrix if $a_{ij} = 0$ when $i > j$.

e.g. $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 5 & 8 \\ 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 9 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.

03. Equality of matrices :

Two matrices A and B are said to be equal and written as A = B, if they are of the **same orders** and their **corresponding elements are identical** *i.e.* $a_{ij} = b_{ij}$ for all i and j.

That is $a_{11} = b_{11}$, $a_{22} = b_{22}$, $a_{23} = b_{23}$, $a_{32} = b_{32}$, $a_{33} = b_{33}$ etc.

04. Addition of matrices :

If A and B are two $m \times n$ matrices, then another $m \times n$ matrix obtained by adding the corresponding elements of the matrices A and B is called the sum of the matrices A and B and is denoted by '$A + B$'.

Thus if $A = [a_{ij}]$, $B = [b_{ij}]$ $\Rightarrow A + B = [a_{ij} + b_{ij}]$.

☑ **Properties of matrix addition :**

(a) Commutative property : $A + B = B + A$

(b) Associative property : $A + (B + C) = (A + B) + C$

(c) Cancellation laws : **(i)** Left cancellation - $A + B = A + C \Rightarrow B = C$

(ii) Right cancellation - $B + A = C + A \Rightarrow B = C$.

(d) Existence of additive identity : For any matrix A, $A + O = O + A = A$; such that the matrix A and null matrix O are of same order. That is, O is the additive identity for matrix addition.

(e) Existence of additive inverse : For any matrix A, $A + (-A) = (-A) + A = O$.
Here $(-A)$ is the additive inverse of A or negative of A.

05. Multiplication of a matrix by a scalar :

If an $m \times n$ matrix A is multiplied by a scalar k (say), then the new kA matrix is obtained by multiplying each element of matrix A by scalar k. Thus if $A = [a_{ij}]$ and it is multiplied by a scalar k then, $kA = [k a_{ij}]$, *i.e.,* $A = [a_{ij}]$ $\therefore kA = [k a_{ij}]$.

e.g. $A = \begin{bmatrix} 2 & -1 \\ 6 & 4 \end{bmatrix}$ $\Rightarrow 3A = 3\begin{bmatrix} 2 & -1 \\ 6 & 4 \end{bmatrix}$

$\therefore 3A = \begin{bmatrix} 6 & -3 \\ 18 & 12 \end{bmatrix}$.

WORKED OUT ILLUSTRATIVE EXAMPLES

Ex01. (a) Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \dfrac{(i + 2j)^2}{2}$.

(b) Write the element a_{32} of a 3×3 matrix $A = (a_{ij})$ where $a_{ij} = \dfrac{2i + j}{4}$.

Sol. **(a)** Consider $A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be the required matrix.

As $a_{ij} = \dfrac{[i+2j]^2}{2}$, so we have $a_{11} = \dfrac{[1+2(1)]^2}{2} = \dfrac{9}{2}$, $a_{12} = \dfrac{25}{2}$, $a_{21} = 8$, $a_{22} = 18$.

So, the required matrix is $A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$.

(b) Since $a_{ij} = \dfrac{2i+j}{4}$

$\Rightarrow a_{32} = \dfrac{2\times 3+2}{4} = \dfrac{8}{4} = 2$.

Ex02. Find the value of a, if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$.

Sol. We have $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

By equality of matrices, we get : $a - b = -1$, $2a + c = 5$, $2a - b = 0$ and $3c + d = 13$.
Solving these equations, we get : $a = 1$.

Ex03. Find the matrix X, if $X+Y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix}$ and $X-Y = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

Sol. We have $X+Y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix}$ and $X-Y = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

On adding these two, we get : $(X+Y)+(X-Y) = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

$\Rightarrow 2X = \begin{pmatrix} 10 & 0 \\ 2 & 8 \end{pmatrix}$

$\therefore X = \begin{pmatrix} 5 & 0 \\ 1 & 4 \end{pmatrix}$.

Ex04. (a) For what value (s) of x, the matrix $\begin{pmatrix} -1 & 0 & y-x \\ 0 & 0 & 0 \\ 0 & x+y-6 & 5 \end{pmatrix}$ is a diagonal matrix?

(b) For what value (s) of $a+x$, the matrix $\begin{pmatrix} 6 & 0 & 0 \\ 0 & 2a+6 & 0 \\ 0 & 0 & x+3 \end{pmatrix}$ is a scalar matrix?

Sol. **(a)** $\because a_{ij} = 0$ if $i \neq j$ for a diagonal matrix so, $y - x = 0$, $x + y - 6 = 0$.
On solving, we get : $x = 3$
(b) $\because a_{ij} = k \ \forall \ i = j$ for a scalar matrix so, $6 = 2a + 6 = x + 3$.
On solving, we get : $a = 0$, $x = 3$.
Therefore, $a + x = 0 + 3 = 3$.

Ex05. (a) If $\begin{bmatrix} 2 & 5 \\ 3 & -7 \end{bmatrix} - A = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$, then find the matrix A.

(b) If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A.

Sol. **(a)** Here $\begin{bmatrix} 2 & 5 \\ 3 & -7 \end{bmatrix} - A = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 3 & -7 \end{bmatrix} - A + A = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} + A$

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 3 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} + A$

$\Rightarrow A = \begin{bmatrix} 2 & 5 \\ 3 & -7 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 2 & 3 \\ 2 & -10 \end{bmatrix}$.

(b) $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow 3A - B + B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow 3A = \begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix}$ $\quad\Rightarrow A = \frac{1}{3} \times \begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$.

Ex06. Find a matrix A, such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

Sol. As $2A - 3B + 5C = O$

$\Rightarrow 2A = 3B - 5C$

$\Rightarrow 2A = \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} - \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix}$

$\Rightarrow 2A = \begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$

$\Rightarrow A = \frac{1}{2}\begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$

$\therefore A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$.

Ex07. Find the values of x and y, if $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - \begin{bmatrix} 5x \\ 6y \end{bmatrix} = 3\begin{bmatrix} -2 \\ -3 \end{bmatrix}$.

Sol. We have $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - \begin{bmatrix} 5x \\ 6y \end{bmatrix} = 3\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x^2 - 5x \\ y^2 - 6y \end{bmatrix} = \begin{bmatrix} -6 \\ -9 \end{bmatrix}$

By equality of matrices, we get : $x^2 - 5x = -6$ and, $y^2 - 6y = -9$

$\Rightarrow x^2 - 5x + 6 = 0 ...(i)$,

$y^2 - 6y + 9 = 0 ...(ii)$

By (i), $x^2 - 5x + 6 = 0 \quad \Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow (x-2)(x-3) = 0$

$\therefore x = 2, 3$

By (ii), $y^2 - 6y + 9 = 0$

$\Rightarrow (y-3)^2 = 0$

$\therefore y = 3$

Therefore, $x = 2, 3;\ y = 3$.

Ex08. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the values of k, a and b. Hence, find $(b)^{a-k}$.

Sol. $\because kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} \qquad \therefore k\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

By equality of matrices, we get : $2k = 3a,\ 3k = 2b,\ -4k = 24$

$\Rightarrow 2(-6) = 3a,\ 3(-6) = 2b,\ k = -6$

$\therefore a = -4,\ b = -9,\ k = -6$.

Also, $(b)^{a-k} = (-9)^{-4+6} = 81$.

EXERCISE 1.1

Q01. If a matrix has 12 elements, what are the possible orders it can have?

Q02. **(a)** How many matrices of order 2×3 are possible with each entry 0 or 1?

(b) What is the number of all possible matrices of order 3×3 with each entry as 0 or 1?

(c) Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3.

Q03. **(a)** Construct a matrix $[a_{ij}]_{4\times 3}$ such that $a_{ij} = \frac{i-j}{i+j}$.

(b) Write a 3×2 matrix B, such that $b_{ij} = \frac{|i-2j|}{3}$.

(c) Construct a 2×3 matrix A whose elements are given by $a_{ij} = \begin{cases} i-2j, & \text{if } i > j \\ i-j, & \text{if } i = j \\ -i+3j, & \text{if } i < j \end{cases}$.

Q04. **(a)** What is the element a_{23} in the matrix $A = \lambda\left[a_{ij}\right]_{3\times 3}$ s.t. $a_{ij} = \frac{2(9i-j)}{3}$?

(b) For a 2×2 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$, write the value of a_{12}.

(c) Write the element a_{23} of a 3×3 matrix $A = (a_{ij})$ whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$.

(d) Write the element a_{12} of the matrix $A=[a_{ij}]_{2\times2}$, whose elements a_{ij} are given by $a_{ij}=e^{2ix}\sin jx$.

Q05. Let $A=\begin{bmatrix}3&0&0\\0&x&0\\0&0&3\end{bmatrix}$. For what value of x, A will be a scalar matrix?

Q06. If $A=\begin{pmatrix}\cos\omega&-\sin\omega\\\sin\omega&\cos\omega\end{pmatrix}$, then for what value of ω is A an identity matrix?

Q07. If $A=\begin{bmatrix}2&4\\3&2\end{bmatrix}$ and $B=\begin{bmatrix}-2&5\\3&4\end{bmatrix}$, then find $(3A-B)$.

Q08. If $A=\text{diag}\begin{pmatrix}1&-1&2\end{pmatrix}$ and $B=\text{diag}\begin{pmatrix}2&3&-1\end{pmatrix}$, find $3A+4B$.

Q09. Simplify : $\cos\omega\begin{bmatrix}\cos\omega&\sin\omega\\-\sin\omega&\cos\omega\end{bmatrix}+\sin\omega\begin{bmatrix}\sin\omega&-\cos\omega\\\cos\omega&\sin\omega\end{bmatrix}$.

Q10. If $A=\begin{bmatrix}8&0\\4&-2\\3&6\end{bmatrix}$ and $B=\begin{bmatrix}2&-2\\4&2\\-5&1\end{bmatrix}$, then find the matrix X, such that $2A+3X=5B$.

Q11. If $\begin{bmatrix}-2&4&-2\\3&7&3\end{bmatrix}+A=\begin{bmatrix}-1&2&6\\4&5&0\end{bmatrix}$, then find the matrix A.

Q12. Solve for the unknown variables viz. w, x, y, z, a, b, c (*as the* ***case*** *may be*) in the followings :

(a) $\begin{bmatrix}x-y&2x+z\\2x-y&3z+a\end{bmatrix}=\begin{bmatrix}-1&5\\0&13\end{bmatrix}$ **(b)** $\begin{bmatrix}7&14\\15&14\end{bmatrix}=2\begin{bmatrix}x&5\\7&y-3\end{bmatrix}+\begin{bmatrix}3&4\\1&2\end{bmatrix}$

(c) $\begin{bmatrix}x^2\\y^2\end{bmatrix}-3\begin{bmatrix}x\\2y\end{bmatrix}=\begin{bmatrix}-2\\9\end{bmatrix}$ **(d)** $\begin{bmatrix}x+3&z+4&2y-7\\4x+6&a-1&0\\b-3&3b&z+2c\end{bmatrix}=\begin{bmatrix}0&6&3y-2\\2x&-3&2c+2\\2b+4&-21&0\end{bmatrix}$

Q13. **(a)** If $2\begin{pmatrix}3&4\\5&x\end{pmatrix}+\begin{pmatrix}1&y\\0&1\end{pmatrix}=\begin{pmatrix}7&0\\10&5\end{pmatrix}$, then find $(x-y)$.

(b) If $A=\begin{pmatrix}0&3\\2&-5\end{pmatrix}$ and $kA=\begin{pmatrix}0&4a\\-8&5b\end{pmatrix}$, find the values of k and a.

(c) Find the value of $(x+y)$ from the following matrix equation :

$$2\begin{bmatrix}x&5\\7&y-3\end{bmatrix}+\begin{bmatrix}3&-4\\1&2\end{bmatrix}=\begin{bmatrix}7&6\\15&14\end{bmatrix}.$$

(d) If $\begin{pmatrix}a+4&3b\\8&-6\end{pmatrix}=\begin{pmatrix}2a+2&b+2\\8&a-8b\end{pmatrix}$, write the value of '$a-2b$'.

EXERCISE 1.2

Q01. **(a)** Find matrix A and B, if $2A-B=\begin{bmatrix}4&-6\\-4&2\end{bmatrix}$ and $A+2B=\begin{bmatrix}-1&0\\1&1\end{bmatrix}$.

(b) Find the matrix A, if $A-B=\begin{bmatrix}1 & 2 & -1\\ 0 & -2 & 1\\ 3 & 2 & -3\end{bmatrix}$ and $A+3B=\begin{bmatrix}0 & 1 & 2\\ -4 & 2 & -2\\ -1 & 0 & -5\end{bmatrix}$.

Q02. Solve for the unknowns x and, y in the followings :

(a) $\begin{bmatrix}x+y & 3\\ 7 & xy\end{bmatrix}=\begin{bmatrix}1 & 3\\ 7 & -12\end{bmatrix}$ **(b)** $\begin{bmatrix}2x+y & 3y\\ 0 & y^2-5y\end{bmatrix}=\begin{bmatrix}x+3 & y^2+2\\ 0 & -6\end{bmatrix}$.

✪ Multiplication of two matrices

01. Def. Let $A=[a_{ij}]$ be a $m\times n$ matrix and $B=[b_{jk}]$ be a $n\times p$ matrix such that the number of columns in A is equal to the number of rows in B, then the $m\times p$ matrix $C=[c_{ik}]$ such that $C_{ik}=\sum_{j=1}^{n}a_{ij}b_{jk}$ is said to be the product of the matrices A and B in that order and it is denoted by AB *i.e.* '$C=AB$'.

e.g. (i) $\begin{pmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{pmatrix}_{2\times3}\begin{pmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{pmatrix}_{3\times2}=\begin{pmatrix}1\times2+(-2)\times4+3\times2 & 1\times3+(-2)\times5+3\times1\\ (-4)\times2+2\times4+5\times2 & (-4)\times3+2\times5+5\times1\end{pmatrix}_{2\times2}=\begin{pmatrix}0 & -4\\ 10 & 3\end{pmatrix}$

(ii) $\begin{pmatrix}2 & -1\\ 3 & 4\end{pmatrix}_{2\times2}\begin{pmatrix}1 & -1 & 0\\ -2 & 1 & -3\end{pmatrix}_{2\times3}=\begin{pmatrix}2\times1+(-1)(-2) & 2(-1)+(-1)\times1 & 2\times0+(-1)(-3)\\ 3\times1+4(-2) & 3(-1)+4\times1 & 3\times0+4(-3)\end{pmatrix}_{2\times3}$

$\Rightarrow \quad =\begin{pmatrix}4 & -3 & 3\\ -5 & 1 & -12\end{pmatrix}$.

☞ For **better illustration**, we need to follow a few more **examples** (to be discussed in the class).

02. Properties of matrix multiplication :

- **(a)** Note that the **product AB is defined *only* when** the number of columns in matrix A is equal to the number of rows in matrix B.
- **(b)** If A and B are $m\times n$ and $n\times p$ matrices respectively, then the matrix AB will be an $m\times p$ matrix *i.e.,* order of matrix AB will be $m\times p$.
- **(c)** In the product AB, A is called the ***pre-factor*** and B is called the ***post-factor***.
- **(d)** If two matrices A and B are such that AB is possible, then it is ***not necessary*** that the product BA is also possible.
- **(e)** If A is a $m\times n$ matrix and both AB as well as BA are defined, then B will be a $n\times m$ matrix.
- **(f)** If A is a $n\times n$ matrix and I_n be the unit matrix of order n, then $A\,I_n=I_n\,A=A$.
- **(g)** ***Existence of multiplicative identity***; for every square matrix A there exists an identity matrix of same order such that $IA=AI=A$.
- **(h)** Matrix multiplication is ***associative*** *i.e.*, $A(BC)=(AB)C$.
- **(i)** Matrix multiplication is ***distributive over the addition*** *i.e.*, $A(B+C)=AB+AC$.
- **(j)** If A and B are diagonal matrices of same order, then we always have $AB=BA$.

03. Powers of a square matrix :

Let A be a square matrix of order n, then AA is defined and it is also a square matrix of order n. That is $A^1=A$, $A^2=AA$, $A^3=A^2A=AA^2=AAA$, …, $A^m=A^{m-1}A=AA^{m-1}$; for all positive integers m.

✪ **Idempotent matrix :** A square matrix A is said to be an idempotent matrix if $\mathbf{A^2 = A}$.

For example, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$.

WORKED OUT ILLUSTRATIVE EXAMPLES

Ex01. If $\mathbf{A = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}}$ **and** $\mathbf{B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}}$ **and** $\mathbf{BA = (b_{ij})}$**, find** $\mathbf{b_{21} + b_{32}}$ **.**

Sol. Here $BA = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix} = \begin{pmatrix} 2\times1+3\times(-4) & 2\times(-2)+3\times2 & 2\times3+3\times5 \\ 4\times1+5\times(-4) & 4\times(-2)+5\times2 & 4\times3+5\times5 \\ 2\times1+1\times(-4) & 2\times(-2)+1\times2 & 2\times3+1\times5 \end{pmatrix}$

Scan me!

$\Rightarrow BA = \begin{pmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{pmatrix} = (b_{ij})$

So, $b_{21} + b_{32} = -16 + (-2) = -18$.

Ex02. Find x, if $\mathbf{[x \quad -5 \quad -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O}$ **.**

Sol. We have $[x \quad -5 \quad -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

$\Rightarrow \left[x\times1+(-5)\times0+(-1)\times2 \quad x\times0+(-5)\times2+(-1)\times0 \quad x\times2+(-5)\times1+(-1)\times3\right]\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

$\Rightarrow \left[x-2 \quad -10 \quad 2x-8\right]\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

$\Rightarrow \left[(x-2)x + (-10)\times4 + (2x-8)\times1\right] = O$

$\Rightarrow \left[x^2 - 48\right] = [0]$

By equality of matrices, we get : $x^2 - 48 = 0$

$\therefore x = \pm 4\sqrt{3}$.

Ex03. (a) Find matrix X, so that $\mathbf{X\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}}$**.**

(b) Find matrix A, such that $\mathbf{\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix}A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}}$**.**

Sol. **(a)** Assume that $P=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}$, $Q=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$.

Since order of P is 2×3 and that of Q is also 2×3 so, matrix X must be of order 2×2.

Let $X=\begin{bmatrix}u & v\\ a & y\end{bmatrix}$ $\{\because X_{2\times2}\,P_{2\times3}=Q_{2\times3}$

$\therefore \begin{bmatrix}u & v\\ a & y\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$

$\Rightarrow \begin{bmatrix}u+4v & 2u+5v & 3u+6v\\ a+4y & 2a+5y & 3a+6y\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$

By equality of matrices, we have :

$u+4v=-7,\ 2u+5v=-8,\ 3u+6v=-9,\ a+4y=2, 2a+5y=4, 3a+6y=6$

On solving these equations simultaneously, we get : $u=1,\ v=-2,\ a=2,\ y=0$.

Hence, $X=\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$.

(b) Let $A=\begin{pmatrix}m & n\\ x & y\end{pmatrix}$.

Now $\begin{pmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{pmatrix}A=\begin{pmatrix}-1 & -8\\ 1 & -2\\ 9 & 22\end{pmatrix}$

$\Rightarrow \begin{pmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{pmatrix}\begin{pmatrix}m & n\\ x & y\end{pmatrix}=\begin{pmatrix}-1 & -8\\ 1 & -2\\ 9 & 22\end{pmatrix}$

$\Rightarrow \begin{pmatrix}2m-x & 2n-y\\ m & n\\ -3m+4x & -3n+4y\end{pmatrix}=\begin{pmatrix}-1 & -8\\ 1 & -2\\ 9 & 22\end{pmatrix}$

By def. of equality of matrices, we get :

$2m-x=-1,\ 2n-y=-8,\ m=1,\ n=-2,\ -3m+4x=9,\ -3n+4y=22$

So, clearly $m=1,\ n=-2,\ x=3,\ y=4$.

Hence, $A=\begin{pmatrix}1 & -2\\ 3 & 4\end{pmatrix}$.

Ex04. If $A=\begin{bmatrix}0 & 6 & 7\\ -6 & 0 & 8\\ 7 & -8 & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 2\\ 1 & -2 & 0\end{bmatrix}$ and $C=\begin{bmatrix}2\\ -2\\ 3\end{bmatrix}$, then verify that $(A+B)C=AC+BC$.

Sol. $A+B=\begin{bmatrix}0 & 6 & 7\\ -6 & 0 & 8\\ 7 & -8 & 0\end{bmatrix}+\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 2\\ 1 & -2 & 0\end{bmatrix}=\begin{bmatrix}0 & 7 & 8\\ -5 & 0 & 10\\ 8 & -10 & 0\end{bmatrix}$

$\therefore (A+B)C=\begin{bmatrix}0 & 7 & 8\\ -5 & 0 & 10\\ 8 & -10 & 0\end{bmatrix}\begin{bmatrix}2\\ -2\\ 3\end{bmatrix}=\begin{bmatrix}0-14+24\\ -10+0+30\\ 16+20+0\end{bmatrix}=\begin{bmatrix}10\\ 20\\ 36\end{bmatrix}$ …(i)

Also $AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$

$\Rightarrow AC = \begin{bmatrix} 0-12+21 \\ -12+0+24 \\ 14+16+0 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$

and, $BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0-2+3 \\ 2+0+6 \\ 2+4+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 6 \end{bmatrix}$

$\Rightarrow AC+BC = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix} + \begin{bmatrix} 1 \\ 8 \\ 6 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 36 \end{bmatrix}$ …(ii)

By (i) and (ii), it is clear that $(A+B)C = AC+BC$.

The property, given in the above example, is the ***Distributive property of matrix addition***.

EXERCISE 1.3

Q01. A matrix X has $a+b$ rows and $a+2$ columns while the matrix Y has $b+1$ rows and $a+3$ columns. Both the matrices XY and YX exist. Find the values of 'a' and 'b'.

Q02. If $(2 \quad 1 \quad 3)\begin{pmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = A$, then write the order of matrix A.

Q03. If $A = [1 \quad 3 \quad 2]$ and $B = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$, then find the matrix AB.

Q04. Give an example of two non-zero 2×2 matrices A and B such that $AB = O$.

Q05. If it is given that $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, then find A^2. Here $i = \sqrt{-1}$.

Q06. If $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then write the matrix A^4.

Q07. If $A = \begin{bmatrix} 0 & 0 \\ -3 & 0 \end{bmatrix}$, then find the value of A^{20}.

Q08. **(a)** Solve the matrix equation : $[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 5 \end{bmatrix} = O$.

(b) Find the values of x, from the matrix equation : $\begin{pmatrix} 3 \\ 2 \end{pmatrix}(1 \quad 5) = \begin{pmatrix} 3 & 7x+y \\ 2y & 10 \end{pmatrix}$.

(c) For what values of x : $[1 \quad 2 \quad 1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$?

(d) Solve for x : $\begin{bmatrix} x & 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 0 \\ 2 & 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = O$.

Q09. Evaluate $1-\omega^2-\kappa\eta$, if $A=\begin{bmatrix} \omega & \kappa \\ \eta & -\omega \end{bmatrix}$ satisfies the equation $A^2 = I$.

Q10. **(a)** If A is a square matrix, such that $A^2 = A$, then what is the value of $(I+A)^3 - 7A$?

(b) If A is a square matrix, such that $A^2 = I$, then find the simplified of $(A-I)^3+(A+I)^3-7A$.

Q11. For what value (s) of x, the matrix product $\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$ equals an identity matrix?

EXERCISE 1.4

Q01. If $A=\begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, $C=\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, then calculate AC, BC and $(A+B)C$.

Also, verify that $(A+B)C = AC+BC$.

This property is known as the ***Distributive property of matrix addition***.

Q02. If it is known that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$, find A.

Q03. Let $A=\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B=\begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix}$ such that $A^2+B^2=(A+B)^2$. Find the value (s) of x and y.

Q04. If $A=\begin{bmatrix} 0 & -\tan\frac{x}{2} \\ \tan\frac{x}{2} & 0 \end{bmatrix}$ and I is an identity matrix, then show that

$(I+A)=(I-A)\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$.

Q05. If $\phi(x)=\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then show that $\phi(x).\phi(y)=\phi(x+y)$.

Q06. Prove that the product of matrices $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $\begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$ is a null matrix, when θ and β differ by an odd integral multiple of $\frac{\pi}{2}$.

Q07. Using $1+\omega+\omega^2=0$ and $\omega^3=1$, show that : $\left(\begin{bmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{bmatrix}+\begin{bmatrix}\omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\\ \omega & \omega^2 & 1\end{bmatrix}\right)\begin{bmatrix}1\\ \omega\\ \omega^2\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$.

The identities $\boldsymbol{1+\omega+\omega^2=0}$ *and* $\boldsymbol{\omega^3=1}$ *are the* ***identities for complex cube root of unity***.

✪ Transpose of a matrix

01. Def. If $A=[a_{ij}]_{m\times n}$ be a matrix of order $m\times n$, then the matrix which can be obtained by interchanging the rows and columns of matrix A is said to be a ***transpose of matrix A***.
The transpose of A is denoted by A' or A^T or A^c *i.e.*, if $A=[a_{ij}]_{m\times n}$ then, $A^T=[a_{ji}]_{n\times m}$.

For example, $\begin{bmatrix}3 & 2 & 0\\ 1 & -2 & 6\end{bmatrix}^T=\begin{bmatrix}3 & 1\\ 2 & -2\\ 0 & 6\end{bmatrix}$; $\begin{bmatrix}1 & 2 & 0 & 5\end{bmatrix}'=\begin{bmatrix}1\\2\\0\\5\end{bmatrix}$.

☑ **Properties of Transpose of matrices :**

(a) $(A+B)^T=A^T+B^T$ **(b)** $(A-B)^T=A^T-B^T$
(c) $(A^T)^T=A$ **(d)** $(kA)^T=kA^T$ where, k is any constant
(e) $(AB)^T=B^TA^T$ **(f)** $(ABC)^T=C^TB^TA^T$

02. Symmetric matrix :
A square matrix $A=[a_{ij}]_{m\times m}$ is said to be a *symmetric matrix* if $A^T=A$.
That is, if $A=[a_{ij}]$ then, $A^T=[a_{ji}]=[a_{ij}]$ $\Rightarrow A^T=A$.

For example, $\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}$, $\begin{bmatrix}2+i & 1 & 3\\ 1 & 2 & 3+2i\\ 3 & 3+2i & 4\end{bmatrix}$.

03. Skew-symmetric matrix :
A square matrix $A=[a_{ij}]$ is said to be a *skew-symmetric matrix* if $A^T=-A$ *i.e.*, if $A=[a_{ij}]$, then $A^T=[a_{ji}]=-[a_{ij}]$ $\Rightarrow A^T=-A$.

For example, $\begin{bmatrix}0 & 1 & -3\\ -1 & 0 & 5\\ 3 & -5 & 0\end{bmatrix}$, $\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}$.

☑ **Facts you should know :**

(a) For a skew symmetric matrix, ***all the diagonal elements are zero***.
(b) The matrices AA^T and A^TA are symmetric matrices.
(c) For any square matrix A, the matrix $A+A^T$ is a symmetric matrix and $A-A^T$ is a skew-symmetric matrix *always*.
(d) Further note that ***any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix*** i.e., $A=\frac{1}{2}(P)+\frac{1}{2}(Q)$, where $P=A+A^T$ is a symmetric matrix and $Q=A-A^T$ is a skew-symmetric matrix.

04. Orthogonal matrix :

A matrix A is said to be orthogonal if $A.A^T = I$, where A^T is transpose of A.

e.g. Let $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, $A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$.

Note that, $A\,A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix} \times \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

So, here matrix A is an **orthogonal matrix**.

☑ For an orthogonal matrix A, we always have **Det.(A) = ±1 i.e., |A| = ±1** (to be discussed **later** in the Determinants topic).

WORKED OUT ILLUSTRATIVE EXAMPLES

Ex01. (a) If A is 2×3 matrix and B is a matrix such that $A'B$ and BA' are both defined. Then what is the order of matrix B?

(b) If $A = \begin{bmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{bmatrix}$, then find $A'A$.

Sol. **(a)** Let order of matrix B be $m \times n$.

We know order of A' will be 3×2.

As $A'B$ is defined so, clearly $2 = m$.

Also BA' is defined so, clearly $n = 3$.

Therefore, order of matrix B is 2×3.

(b) We have $A = \begin{bmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{bmatrix}$

$\Rightarrow A' = \begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix}$.

$\therefore A'A = \begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix}\begin{bmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{bmatrix}$

$\Rightarrow A'A = \begin{bmatrix} \sin^2 x + \cos^2 x & \sin x\cos x - \cos x \sin x \\ \cos x \sin x - \sin x \cos x & \cos^2 x + \sin^2 x \end{bmatrix}$

Therefore, $A'A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$.

Ex02. If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, find α satisfying $0 < \alpha < \frac{\pi}{2}$ when $A + A^T = \sqrt{2}\, I_2$; where A^T is the transpose of A.

Sol. Here $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

$\Rightarrow A^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

Since $A + A^T = \sqrt{2}\, I_2$

$$\therefore \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} + \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \sqrt{2}\ I$$

$$\Rightarrow \begin{pmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{pmatrix} = \sqrt{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$$

By def. of equality of matrices, we get : $2\cos\alpha = \sqrt{2}$

$\Rightarrow \cos\alpha = \dfrac{1}{\sqrt{2}}$ $\qquad \therefore \alpha = \dfrac{\pi}{4}$.

Ex03. Show that all the diagonal elements of a skew symmetric matrix are zero.

Sol. Let $A = [a_{ij}]$ be a square matrix such that it is skew-symmetric.

So, $A = -A^T$.

That is, $[a_{ij}] = -[a_{ji}]$.

For its diagonal elements, we have $[a_{ii}] = -[a_{ii}]$ which implies, $2[a_{ii}] = O$

(Replacing j by i)

$\Rightarrow [a_{ii}] = [0]$

$\therefore a_{ii} = 0$.

Hence, all the diagonal elements of a skew symmetric matrix are zero.

Ex04. Express $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Sol. For the matrix $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$, we have $A^T = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}$.

$$\therefore A + A^T = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 10 & 7 \\ 5 & 7 & 10 \end{bmatrix} \text{ and } A - A^T = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

Let $P = \frac{1}{2}(A + A^T) = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix}$ and, $Q = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}$.

We observe that, $P^T = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix}^T = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} = P$

$\therefore$ P is symmetric matrix.

Further observe that, $Q^T = \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 & -1/2 \\ -1 & 0 & 1/2 \\ 1/2 & -1/2 & 0 \end{bmatrix}$

$$\Rightarrow Q^T = -\begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix} = -Q$$

$\therefore$ Q is skew-symmetric matrix.

Hence we have, $P+Q=\begin{bmatrix}3 & 3 & 5/2\\ 3 & 5 & 7/2\\ 5/2 & 7/2 & 5\end{bmatrix}+\begin{bmatrix}0 & -1 & 1/2\\ 1 & 0 & -1/2\\ -1/2 & 1/2 & 0\end{bmatrix}=\begin{bmatrix}3 & 2 & 3\\ 4 & 5 & 3\\ 2 & 4 & 5\end{bmatrix}=A$.

Thus, we have expressed matrix A as the sum of a symmetric matrix and a skew-symmetric matrix.

Ex05. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute. (A and B *commute* means $AB=BA$).

Sol. Given that A and B are both symmetric matrices.

$\therefore A=A^T$ and $B=B^T$ ...(i)

Let $P=AB$

$\Rightarrow P^T=(AB)^T$

$\Rightarrow P^T=B^TA^T$

$\Rightarrow P^T=BA$ (By (i))

If A and B commute then, $AB=BA$

$\therefore P^T=AB$ i.e., $P^T=P$.

So, P is symmetric matrix.

Ex06. If the matrix $A=\begin{bmatrix}0 & a & -3\\ 2 & 0 & -1\\ b & 1 & 0\end{bmatrix}$ is skew symmetric, find the value of 'a' and 'b'.

Sol. As A is skew symmetric so, $A=-A^T$ i.e., $a_{ij}=-a_{ji}$, if $A=[a_{ij}]$.

Therefore, $a_{12}=-a_{21}\Rightarrow a=-2$

and, $a_{31}=-a_{13}\Rightarrow b=-(-3)=3$.

Hence value of a is '–2' and value of b is '3'.

Ex07. If $A=\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & x\\ -2 & 2 & -1\end{pmatrix}$ is matrix satisfying $AA'=9I$, find x.

Sol. As $AA'=9I$

$$\Rightarrow \begin{pmatrix}1 & 2 & 2\\ 2 & 1 & x\\ -2 & 2 & -1\end{pmatrix}\begin{pmatrix}1 & 2 & -2\\ 2 & 1 & 2\\ 2 & x & -1\end{pmatrix}=9\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}9 & 4+2x & 0\\ 4+2x & 5+x^2 & -2-x\\ 0 & -2-x & 9\end{pmatrix}=\begin{pmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{pmatrix}$$

By equality of matrices, we get : $4+2x=0,\ 5+x^2=9,\ -2-x=0$

On solving these, we've : $x=-2$ (which satisfies the given condition).

EXERCISE 1.5

Q01. If $A=\begin{pmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{pmatrix}$ and $A+A^T=I_2$, then what is the value of x?

Q02. If $A=\begin{bmatrix}\sin x & \cos x\\ -\cos x & \sin x\end{bmatrix}$, then verify that $A'A=I$.

Q03. **(a)** If A is a matrix of 2×3 and B is of 3×5, what is the order of $(AB)^T$?

(b) If A is 3×4 matrix and B is a matrix such that A^TB and BA^T are both defined, then what is the order of matrix B?

Q04. **(a)** Write the values of 'p' and 'q' such that the matrix $A=\begin{pmatrix}0 & 5 & -3\\ -5 & p & 4\\ q & -4 & 0\end{pmatrix}$ is skew symmetric.

(b) Matrix $A=\begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}$ is given to be symmetric, find the values of 'a' and 'b'.

Q05. Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

Q06. If A and B are symmetric matrices, prove that $AB-BA$ is a skew-symmetric matrix.

Q07. Show that the matrix B^TAB is symmetric or skew-symmetric according as A is the symmetric or skew-symmetric.

Q08. If B is skew-symmetric matrix, write whether ABA' is symmetric or skew-symmetric.

Q09. Show that the elements on the main diagonal of a skew-symmetric matrix are all zero.

Q10. If the matrix $A=\begin{pmatrix}3 & 5\\ 7 & 9\end{pmatrix}$ is written as $A=P+Q$, where P is a symmetric matrix and Q is skew symmetric matrix, then write the matrix P.

Q11. When the matrix $A=\begin{pmatrix}1 & 0 & 0\\ -1 & 0 & 5\\ 2 & 6 & 8\end{pmatrix}$ is written as $A=P+Q$, where P is a symmetric matrix and Q is a skew-symmetric matrix, then write the matrix Q.

EXERCISE 1.6

Q01. If $A=\begin{bmatrix}2 & -1 & 2\\ -4 & 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & -1\\ 1 & 3\\ 4 & 5\end{bmatrix}$, then verify that $(AB)^T=B^TA^T$.

Q02. If $A=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$ and $B'=\begin{bmatrix}0 & -1 & 1\\ 3 & 2 & 1\end{bmatrix}$, then find $(BA)'$.

Q03. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$ is a matrix satisfying $AA^T=9I_3$, then find the values of 'a' and 'b'.

Q04. Define a symmetric and skew-symmetric matrix.

Prove that for the matrix X, $X-X^T$ is skew-symmetric matrix whereas $X+X^T$, XX^T and X^TX is symmetric matrix, where $X=\begin{bmatrix}-1 & 1\\ 2 & -4\end{bmatrix}$.

Q05. If $A=\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & -1\\ -1 & 3\end{bmatrix}$, show that AB – BA is a skew-symmetric matrix.

Q06. Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, where $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$.

Q07. Express the matrix $\begin{bmatrix}2 & -1\\ 4 & 5\end{bmatrix}$, as the sum of symmetric and skew-symmetric matrix.

Q08. Express the matrix $\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

Q09. If l_i, m_i, n_i; $i=1, 2, 3$ denote the direction cosines of three mutually perpendicular lines in the space, then prove that $AA^T=I$ such that $A=\begin{bmatrix}l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\\ l_3 & m_3 & n_3\end{bmatrix}$.

Above Question - Q09 is based on the Concept from ***Three Dimensional Geometry****, NCERT Part II.*

✪ Invertible Matrices

01. Def. If A is a square matrix of order *m* and if there exists another square matrix B of the same order *m*, such that AB = BA = I, then B is called the inverse matrix of A and it is denoted by A^{-1}.
A matrix having an inverse is said to be ***invertible***.

> *It is to note that if B is inverse of A, then A is also the inverse of B.*
> *In other words, if it is known that* $AB = BA = I$, *then* $A^{-1} = B \Leftrightarrow B^{-1} = A$.

02. Inverse or Reciprocal of a square matrix :
If A is a square matrix of order n, then a matrix B (if such a matrix exists) is called the inverse of A if $AB = BA = I_n$.
Also note that the inverse of a square matrix A is denoted by A^{-1} and we write, $A^{-1} = B$.

> • *Inverse of a square matrix A exists if and only if A is non-singular matrix i.e.,* $|A| \neq 0$ *(explained later in the Determinant section).*
> • *If B is inverse of A, then A is also the inverse of B.*

☑ Inverse of a square matrix, if it exists, is unique (**Uniqueness of inverse of matrices**).
☑ If A and B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1}A^{-1}$.

03. Matrix polynomial :
Let $f(x)=a_0x^m+a_1x^{m-1}+a_2x^{m-2}...+a_{m-1}x+a_m$ be a polynomial in variable x and A be a square matrix of order n, then $f(A)=a_0A^m+a_1A^{m-1}+a_2A^{m-2}+...+a_{m-1}A+a_m$ is called a matrix polynomial in A. Thus, to obtain f(A), just replace x by A in f(x) and the constant term is multiplied by the identity matrix (unit matrix) of the order same as that of A i.e., $f(A)=a_0A^m+a_1A^{m-1}+...+a_{m-1}A+a_mI$.
The polynomial equation $f(x)=0$ is said to be satisfied by the matrix A, if $f(A)=O$, where O is the null matrix of the order same as that of A.

WORKED OUT ILLUSTRATIVE EXAMPLES

Ex01. If $A=\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}$, then find A^2-5A.

Sol. $A^2=A.A=\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}$

$\therefore A^2-5A=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}-\begin{bmatrix}10&0&5\\10&5&15\\5&-5&0\end{bmatrix}$

Therefore, $A^2-5A=\begin{bmatrix}-5&-1&-3\\-1&-7&-10\\-5&4&-2\end{bmatrix}$.

Ex02. If $A=\begin{bmatrix}4&2\\-1&1\end{bmatrix}$, show that $(A-2I)(A-3I)=O$.

Sol. LHS : $(A-2I)(A-3I)=\left\{\begin{bmatrix}4&2\\-1&1\end{bmatrix}-2\begin{bmatrix}1&0\\0&1\end{bmatrix}\right\}\left\{\begin{bmatrix}4&2\\-1&1\end{bmatrix}-3\begin{bmatrix}1&0\\0&1\end{bmatrix}\right\}$

$=\begin{bmatrix}2&2\\-1&-1\end{bmatrix}\begin{bmatrix}1&2\\-1&-2\end{bmatrix}$

$=\begin{bmatrix}0&0\\0&0\end{bmatrix}=O=RHS$.

Ex03. If $A=\begin{bmatrix}2&-1\\-1&2\end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2=4A-3I$. Hence, find A^{-1}.

Sol. We have $A=\begin{bmatrix}2&-1\\-1&2\end{bmatrix}$

$\Rightarrow A^2=A.A=\begin{bmatrix}2&-1\\-1&2\end{bmatrix}\begin{bmatrix}2&-1\\-1&2\end{bmatrix}=\begin{bmatrix}5&-4\\-4&5\end{bmatrix}$ ...(i)

Also $4A-3I=4\begin{bmatrix}2&-1\\-1&2\end{bmatrix}-3\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}8&-4\\-4&8\end{bmatrix}-\begin{bmatrix}3&0\\0&3\end{bmatrix}=\begin{bmatrix}5&-4\\-4&5\end{bmatrix}$...(ii)

By (i) and (ii), we get : $A^2=4A-3I$.

Pre-multiplying both sides by A^{-1} we get : $A^{-1}AA=4A^{-1}A-3A^{-1}I$

$\Rightarrow IA=4I-3A^{-1}$

$\Rightarrow 3A^{-1}=4I-A=4\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}2&-1\\-1&2\end{bmatrix}=\begin{bmatrix}2&1\\1&2\end{bmatrix}$

$\therefore A^{-1}=\frac{1}{3}\begin{bmatrix}2&1\\1&2\end{bmatrix}$ or, $A^{-1}=\begin{bmatrix}2/3&1/3\\1/3&2/3\end{bmatrix}$.

Ex04. If $A=\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}$ and $A^2-\lambda A+\mu I=O$, then find the values of λ and μ.

Sol. We have $A=\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}$

$$\therefore A^2=AA=\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}=\begin{pmatrix}2\times2+(-1)\times(-1) & 2\times(-1)+(-1)\times2\\ (-1)\times2+2\times(-1) & (-1)\times(-1)+2\times2\end{pmatrix}$$

$$\Rightarrow A^2=\begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}$$

Now we also have $A^2-\lambda A+\mu I=O$

$$\therefore \begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}-\lambda\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}+\mu\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}-\begin{pmatrix}2\lambda & -\lambda\\ -\lambda & 2\lambda\end{pmatrix}+\begin{pmatrix}\mu & 0\\ 0 & \mu\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}5-2\lambda+\mu & -4+\lambda\\ -4+\lambda & 5-2\lambda+\mu\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

By equality of matrices, we get : $5-2\lambda+\mu=0,\ -4+\lambda=0$

$\Rightarrow \mu=2\lambda-5,\ \lambda=4$

Hence, $\lambda=4$ and $\mu=3$.

EXERCISE 1.7

Q01. Find the matrix A, if it is given that $\begin{bmatrix}2 & 0\\ 3 & 2\end{bmatrix}A=A^2$.

Q02. If $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ and $B=\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ such that $AB=BA=I$, then write A^{-1}.

Q03. If $A=\begin{bmatrix}1 & 3\\ 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & -3\\ 0 & 1\end{bmatrix}$, then write the inverse of matrix A.

Q04. Let $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$. Then write the inverse of B.

Q05. Given that A and B are invertible matrices of the same order; such that $(AB)^{-1}=B^{-1}A^k$. Then find the value of k.

EXERCISE 1.8

Q01. Find the value of x, y and z, if $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfies $A'=A^{-1}$.

OR Find the values of x, y, z; if the matrix $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfies the equation $A^TA=I$.

Q02. Show that $\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$ satisfies the equation $x^2-3x-7=0$.

Thus find the inverse of given matrix.

Q03. **(a)** If $A=\begin{bmatrix}3 & 2 & 0\\ 1 & 4 & 0\\ 0 & 0 & 5\end{bmatrix}$ then, prove that $(A-5I)(A-2I)=O$. Hence, find A^{-1}.

(b) For the matrix $A=\begin{pmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{pmatrix}$, show that $A^2-5A+4I=O$. Hence, find A^{-1}.

Q04. For the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$, verify that $A^3-6A^2+5A+11\,I=O$. Hence, find A^{-1}.

Q05. If $A=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$, then find x and y so that $A^2+xA+yI=O$. Hence, find A^{-1}.

MISCELLANEOUS EXAMPLES

Ex01. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, then show that $A^2-5A+7I=O$.

Sol. We have $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$\therefore A^2=A.A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$\Rightarrow A^2=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$ …(i)

$-5A=-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}=\begin{bmatrix}-15 & -5\\ 5 & -10\end{bmatrix}$ …(ii)

And, $7I=7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$ …(iii)

Adding these three equations, we get : $A^2-5A+7I=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}+\begin{bmatrix}-15 & -5\\ 5 & -10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

$\Rightarrow A^2-5A+7I=\begin{bmatrix}8-15+7 & 5-5+0\\ -5+5+0 & 3-10+7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$\therefore A^2-5A+7I=O$.

Ex02. If $A=\begin{pmatrix}1&0&2\\0&2&1\\2&0&3\end{pmatrix}$ and $A^3-6A^2+7A+kI_3=O$, find k.

Sol. We have $A=\begin{pmatrix}1&0&2\\0&2&1\\2&0&3\end{pmatrix}$

$$\Rightarrow A^2=AA=\begin{pmatrix}1&0&2\\0&2&1\\2&0&3\end{pmatrix}\begin{pmatrix}1&0&2\\0&2&1\\2&0&3\end{pmatrix}=\begin{pmatrix}5&0&8\\2&4&5\\8&0&13\end{pmatrix}$$

Also, $A^3=AA^2=\begin{pmatrix}1&0&2\\0&2&1\\2&0&3\end{pmatrix}\begin{pmatrix}5&0&8\\2&4&5\\8&0&13\end{pmatrix}=\begin{pmatrix}21&0&34\\12&8&23\\34&0&55\end{pmatrix}$

Now $A^3-6A^2+7A+kI_3=O$

$$\Rightarrow\begin{pmatrix}21&0&34\\12&8&23\\34&0&55\end{pmatrix}-6\begin{pmatrix}5&0&8\\2&4&5\\8&0&13\end{pmatrix}+7\begin{pmatrix}1&0&2\\0&2&1\\2&0&3\end{pmatrix}+k\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=O$$

$$\Rightarrow\begin{pmatrix}21&0&34\\12&8&23\\34&0&55\end{pmatrix}-\begin{pmatrix}30&0&48\\12&24&30\\48&0&78\end{pmatrix}+\begin{pmatrix}7&0&14\\0&14&7\\14&0&21\end{pmatrix}+\begin{pmatrix}k&0&0\\0&k&0\\0&0&k\end{pmatrix}=O$$

$$\Rightarrow\begin{pmatrix}-2&0&0\\0&-2&0\\0&0&-2\end{pmatrix}+\begin{pmatrix}k&0&0\\0&k&0\\0&0&k\end{pmatrix}=O$$

$$\Rightarrow\begin{pmatrix}k-2&0&0\\0&k-2&0\\0&0&k-2\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$$

By equality of matrices, we get : $k-2=0$

$\therefore k=2$.

Ex03. If $A=\begin{bmatrix}3&-4\\1&-1\end{bmatrix}$, then show that $A^n=\begin{bmatrix}1+2n&-4n\\n&1-2n\end{bmatrix}$ $\forall\ n\in N$.

Sol. We shall be using principle of mathematical induction to prove this.

Let $P(n):\ A^n=\begin{bmatrix}1+2n&-4n\\n&1-2n\end{bmatrix}\ \forall n\in N$

For n = 1, $P(1):\ A^1=\begin{bmatrix}1+2(1)&-4(1)\\1&1-2(1)\end{bmatrix}=\begin{bmatrix}3&-4\\1&-1\end{bmatrix}=A$ (Given $A=\begin{bmatrix}3&-4\\1&-1\end{bmatrix}$

$\therefore$ P(1) is true.

Assume that P(k) is true for $k\in N$ i.e., $P(k):\ A^k=\begin{bmatrix}1+2k&-4k\\k&1-2k\end{bmatrix}\ \forall k\in N\ldots$(i)

We have to show that $P(k+1)$ is also true whenever P(k) is true i.e.,

$$P(k+1):\ A^{k+1}=\begin{bmatrix}1+2(k+1) & -4(k+1)\\ k+1 & 1-2(k+1)\end{bmatrix}=\begin{bmatrix}3+2k & -4-4k\\ k+1 & -1-2k\end{bmatrix}$$

Consider LHS : $A^{k+1}=A^kA=\begin{bmatrix}1+2k & -4k\\ k & 1-2k\end{bmatrix}\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ (By (i)

$$=\begin{bmatrix}3+6k-4k & -4-8k+4k\\ 3k+1-2k & -4k-1+2k\end{bmatrix}$$

$$=\begin{bmatrix}3+2k & -4-4k\\ k+1 & -1-2k\end{bmatrix}=\text{RHS.}$$

$\therefore P(k+1)$ is also true.

Hence by principle of mathematical induction, P(n) is always true for all natural numbers n.

Ex04. Let $A=\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}$, then show that $A^2-4A+7I=O$. Using this result, calculate A^3 also.

Sol. Here $A=\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}$

$$\Rightarrow A^2=A.A=\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}=\begin{pmatrix}1 & 12\\ -4 & 1\end{pmatrix}$$

$$\Rightarrow A^2-4A+7I=\begin{pmatrix}1 & 12\\ -4 & 1\end{pmatrix}-4\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}+7\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\therefore A^2-4A+7I=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}=O.$$

Now $A^2-4A+7I=O$

$\Rightarrow A^2=4A-7I$

$\Rightarrow A.A^2=A(4A-7I)$ (Pre-multiplying by A both sides

$\Rightarrow A^3=4A^2-7AI=4(4A-7I)-7A$ (Using $A^2=4A-7I$

$\therefore A^3=16A-28I-7A=9A-28I$

Therefore, $A^3=9A-28I=\begin{pmatrix}18 & 27\\ -9 & 18\end{pmatrix}-\begin{pmatrix}28 & 0\\ 0 & 28\end{pmatrix}$

$$\Rightarrow A^3=\begin{pmatrix}-10 & 27\\ -9 & -10\end{pmatrix}.$$

Ex05. If $A=\begin{pmatrix}1 & -2\\ 2 & -1\end{pmatrix}$, then find A^2+3I. Hence, find A^4.

Sol. As $A^2=A.A=\begin{pmatrix}1 & -2\\ 2 & -1\end{pmatrix}\begin{pmatrix}1 & -2\\ 2 & -1\end{pmatrix}=\begin{pmatrix}-3 & 0\\ 0 & -3\end{pmatrix}$

Therefore, $A^2+3I=\begin{pmatrix}-3 & 0\\ 0 & -3\end{pmatrix}+3\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$$\Rightarrow A^2+3I=\begin{pmatrix}-3 & 0\\ 0 & -3\end{pmatrix}+\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

$\therefore A^2+3I=O.$

Now $A^2+3I=O$ $\Rightarrow A^2=-3I$

$\because A^4 = A^2 . A^2$

$\therefore A^4 = (-3I).(-3I) = 9I.I = 9I$

Hence, $A^4 = \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix}$.

Ex06. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, find A^2 and show that $A^2 = A^{-1}$.

Sol. $A^2 = AA = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

Note that, $A^2A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

Now $A^2A = I$

Post-multiplying both sides by A^{-1}, we get : $A^2AA^{-1} = IA^{-1}$

$\Rightarrow A^2I = A^{-1} \quad \therefore A^2 = A^{-1}$.

Ex07. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the value of 'k' so that $A^2 = kA - 2I$.

Sol. For $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, $A^2 = A.A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

$\therefore A^2 = kA - 2I$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -4 \end{bmatrix}$

By equality of matrices, $4k = 4 \quad \therefore k = 1$.

EXERCISE 1.9

Q01. **(a)** If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, find the value of k.

(b) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, show that $A^{-1} = \frac{1}{19}A$.

Q02. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ and $f(x) = x^2 - 5x - 6$, then find $f(A)$.

Q03. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ and $f(x) = x^2 - 5x + 6$, then find $f(A)$.

OR If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$, find $A^2 - 5A + 16I$. *This sum is **similar** to the question given above.*

Q04. If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, then show that A is a root of the cubic equation $x^3-6x^2+7x+2=0$.

Q05. If $A=\begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}$, then prove that $A^3-23A-40I=O$.

Q06. Let $A=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$ and $f(x)=x^2-4x+7$. Show that $f(A)=O$. Use this result to find A^5.

Q07. If $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$ and $A^2-2B+7I=O$, then find the matrix B.

Q08. If $A=\begin{pmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{pmatrix}$, find $A^2-5A+4I$ and hence find a matrix X s.t. $A^2-5A+4I+X=O$.

Q09. Prove the followings by the **Principle of Mathematical Induction** :

(a) $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$, $n\in N$ if $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$.

(b) $(aI+bA)^n=a^nI+na^{n-1}bA\ \forall\ n\in N$, where I is the identity matrix of 2^{nd} order, if it is given that matrix $A=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$.

(c) $A^n=\begin{bmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\end{bmatrix}$, $n\in N$ if $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$.

(d) $A^n=\begin{bmatrix}a^n & \dfrac{b(a^n-1)}{a-1}\\ 0 & 1\end{bmatrix}$, $n\in Z^+$ if $A=\begin{bmatrix}a & b\\ 0 & 1\end{bmatrix}$ where $a\neq 1$.

Q10. **(a)** If $A=\text{diag}(a\quad b\quad c)$, show that $A^n=\text{diag}(a^n\quad b^n\quad c^n)$ for all positive integers n.

(b) If A and B are square matrices of the same order such that AB = BA, then prove by using induction that $AB^n=B^nA$. Further, prove that $(AB)^n=A^nB^n$ for all $n\in N$.

EXERCISE 1.10

Q01. In XII class examination, 25 students from school A and 35 students from school B appeared. Only 20 students from each school could get through the examination. Out of them, 15 students from school A and 10 students from school B secured full marks. Write down this information in matrix from.

Q02. Let matrix $A=\begin{bmatrix}8 & 16\\ 32 & 48\end{bmatrix}$, where first row represents the number of table fans and second row represents the number of ceiling fans which two manufacturing units x and y makes in one day. Compute 7A and, state what does it represents?

Q03. In Chennai, there are 50 colleges. Each has 30 teachers, 20 non-teaching staffs, 1 Principal, 2 Vice Principals and, 5 peons. Express this information in the form of a column matrix.
Using scalar multiplication, find the total number of posts of each type in the colleges.

Q04. A logistics company stores three products (A, B, and C) in two warehouses (W_1 and W_2). The stock levels are represented as: $X=\begin{bmatrix}100 & 200 & 150\\ 120 & 250 & 180\end{bmatrix}$.

A new shipment of products arrives, adding the following stock: $Y=\begin{bmatrix}30 & 50 & 40\\ 20 & 60 & 50\end{bmatrix}$.

Find the updated stock levels after adding the shipment.

Q05. A company sells three products (P_1, P_2 and P_3) in two cities (C_1 and C_2). The price per unit of each product is: $P=\begin{bmatrix}10 & 15 & 20\end{bmatrix}$.

The number of units sold in each city is: $Q=\begin{bmatrix}100 & 120\\ 150 & 130\\ 200 & 180\end{bmatrix}$.

Find the total revenue generated from each city.

Q06. A city has three major roads (R_1, R_2 and R_3) connecting two traffic points (A and B). The number of vehicles (in hundreds) travelling on these roads on two different days is given by:

$$T_1=\begin{bmatrix}50 & 60 & 40\\ 30 & 20 & 10\end{bmatrix}$$

where Row 1 represents traffic on Monday, Row 2 represents traffic on Tuesday and Columns represent roads R_1, R_2 and R_3.
Due to road maintenance, the traffic on each road decreases by 10% the following week. What will be the new traffic matrix?

Q07. A company has two teams (A and B) working on three different projects (P1, P2, P3). The number of hours each team spends on each project in a week is given by $H=\begin{bmatrix}10 & 15 & 20\\ 12 & 18 & 25\end{bmatrix}$.

Each team member is paid at different hourly rates (in ₹) given as in the matrix $R=\begin{bmatrix}500\\ 550\\ 600\end{bmatrix}$.

Find the total salary paid to each team.

Q08. Three vehicles (namely V_1, V_2 and V_3) use three types of fuels (Petrol, Diesel and Gas). Their daily consumption (in liters) is given in the matrix $F=\begin{bmatrix}5 & 10 & 8\\ 6 & 12 & 7\\ 7 & 9 & 10\end{bmatrix}$. The cost per liter for each fuel type is $C=\begin{bmatrix}100\\ 90\\ 80\end{bmatrix}$. Find the total daily fuel cost for each vehicle.

✪ DETERMINANTS, ITS PROPERTIES & APPLICATIONS

INTRODUCTION

The study of determinants is linked with the study of algebra of matrices. Determinants have many important applications. They are used to find the inverse of a matrix, to obtain the area of a triangle, to verify whether three given points are collinear, to solve system of linear equations etc. Later on, in the chapter **Vector Algebra**, we shall use determinants to find **vector product of two vectors**.

IMPORTANT TERMS, DEFINITIONS & RESULTS

01. Determinants, Minors & Cofactors :

(a) Determinant :

A unique number (real or complex) can be associated to every square matrix $A=[a_{ij}]$ of order m.

This number is called the determinant of the square matrix A, where $a_{ij}=(i,\ j)^{th}$ element of A.

For instance, if $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then, determinant of matrix A is written as $|A|=\begin{vmatrix} a & b \\ c & d \end{vmatrix}=\det.(A)$ and its value is given by "**ad − bc**".

For memorization, $|A|=\begin{vmatrix} a & b \\ c & d \end{vmatrix}=ad-bc$.

- ☑ For a square matrix A, $|A|$ is read as '**determinant of A**' and not modulus of A.
- ☑ Only square matrices have determinants.
- ☑ For a matrix $A=[x]$ of order 1×1, its determinant value is $|A|=x$.

(b) Minors :

Minor of an element a_{ij} of a determinant *(or a determinant corresponding to matrix A)* is the determinant obtained by deleting its i^{th} row and j^{th} column in which a_{ij} lies. Minor of the element a_{ij} is denoted by M_{ij}.

Hence, we can get 9 minors corresponding to the 9 elements of a third order (i.e., 3×3) determinant.

(c) Cofactors :

Cofactor of an element a_{ij}, denoted by A_{ij}, is defined by, $A_{ij}=(-1)^{i+j}M_{ij}$, where M_{ij} is minor of a_{ij}.

Sometimes C_{ij} is used in place of A_{ij} to denote the cofactor of element a_{ij}.

02. Expanding a Determinant :

A determinant can be expanded along any row (or, column) as follows -

Adding the Products of each element of any row (or, column) with its corresponding Cofactor, gives the value of the determinant.

e.g. $\begin{vmatrix} 1 & -2 & -1 \\ 4 & 5 & 0 \\ -3 & 2 & 3 \end{vmatrix}=\{1(5\times3-2\times0)\}+\{(-2)[-(3\times4-0\times(-3))]\}+\{(-1)(4\times2-5\times(-3))\}$

$\Rightarrow\quad =15+24-23=16$. (Note that we have expanded this Det. along first row.

03. Properties of Determinants :

(a) If any two rows or columns of a determinant are *proportional* or *identical*, then its value is equal to *zero*.

e.g. $\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix}=0$

Note that R_1 and R_3 are the same (identical).

(b) The value of a determinant remains *unchanged* if its rows and columns are *interchanged*.

e.g. $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$.

Here rows and columns have been interchanged, but there is *no effect* on the value of determinant. In other words, the value of determinant of matrix A and its transpose A^T remains the same.

(c) If each element of a row or a column of a determinant is multiplied by a constant *k*, then the value of new determinant is *k* times the value of the original determinant.

e.g. $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \Rightarrow \Delta_1 = k\Delta$.

(d) If *any two* rows or columns are *interchanged*, then the determinant retains its *absolute* value, but its *sign* is changed.

e.g. $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} a_3 & b_3 & c_3 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix} \Rightarrow \Delta_1 = -\Delta$ (Here $R_1 \leftrightarrow R_3$

(e) If every element of some column or row is the *sum* of two terms, then the determinant is equal to the sum of two determinants; one containing only the first term in place of each sum, the other only the second term. The remaining elements of both determinants are the same as given in the original determinant.

e.g. $\Delta = \begin{vmatrix} a_1+\alpha & b_1 & c_1 \\ a_2+\beta & b_2 & c_2 \\ a_3+\gamma & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha & b_1 & c_1 \\ \beta & b_2 & c_2 \\ \gamma & b_3 & c_3 \end{vmatrix}$.

04. Area of triangle :

Area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by,

$$\Delta = \text{Magnitude of } \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ (in Sq. units)}$$

(i) Since area is a positive quantity, we take **absolute value of the determinant** Δ shown above.

(ii) If the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are **collinear** then $\Delta = 0$.

(iii) The **equation of a line** passing through the points (x_1, y_1) and (x_2, y_2) can be obtained by the expression given here : $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$.

05. Product of Two Determinants :

Let $\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$.

$$\therefore \Delta_1\Delta_2 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}\begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$$

$$\Rightarrow \Delta_1\Delta_2 = \begin{vmatrix} a_1\alpha_1 + b_1\beta_1 + c_1\gamma_1 & a_1\alpha_2 + b_1\beta_2 + c_1\gamma_2 & a_1\alpha_3 + b_1\beta_3 + c_1\gamma_3 \\ a_2\alpha_1 + b_2\beta_1 + c_2\gamma_1 & a_2\alpha_2 + b_2\beta_2 + c_2\gamma_2 & a_2\alpha_3 + b_2\beta_3 + c_2\gamma_3 \\ a_3\alpha_1 + b_3\beta_1 + c_3\gamma_1 & a_3\alpha_2 + b_3\beta_2 + c_3\gamma_2 & a_3\alpha_3 + b_3\beta_3 + c_3\gamma_3 \end{vmatrix}.$$

Here we have multiplied **rows by rows**. We can also multiply **rows by columns** or, **columns by rows** or, **columns by columns**. It's because of the fact that $|A| = |A^T|$.

Moreover $|AB| = |A||B| = |B||A| = |BA|$, where A and B are square matrices of the same order.

06. Derivative of a Determinant :

Let $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ p(x) & q(x) & r(x) \\ s(x) & t(x) & u(x) \end{vmatrix}$.

$$\therefore \Delta'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ p(x) & q(x) & r(x) \\ s(x) & t(x) & u(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ p'(x) & q'(x) & r'(x) \\ s(x) & t(x) & u(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ p(x) & q(x) & r(x) \\ s'(x) & t'(x) & u'(x) \end{vmatrix}.$$

Here we have differentiated through **rows**. Same can be done through **columns** too.
Alternatively, we can use properties of determinants to expand the determinant and, then differentiate it.

WORKED OUT ILLUSTRATIVE EXAMPLES

Ex01. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

Sol. We have $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow (x+1)(x+2) - (x-1)(x-3) = 4 \times 3 - (-1) \times 1$

$\Rightarrow (x^2 + 3x + 2) - (x^2 - 4x + 3) = 12 + 1$

$\Rightarrow 7x - 1 = 13$

$\therefore x = 2$.

Ex02. If $A = [a_{ij}]$ is a matrix of order 2×2, such that $|A| = -15$ and C_{ij} represents the cofactor of a_{ij}, then find $a_{21}C_{21} + a_{22}C_{22}$.

Sol. As $A = [a_{ij}]_{2\times2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

Consider $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

On expanding along R_2, $|A| = a_{21}C_{21} + a_{22}C_{22}$

$\therefore |A| = -15$.

Ex03. If the points (2, –3), (k, –1) and (0, 4) are collinear, then find the value of k.

Sol. The points (2, –3), (k, –1) and (0, 4) are collinear.

So we must have, $\begin{vmatrix} 2 & -3 & 1 \\ k & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0$

Expanding along third row, we get $0(-3+1)-4(2-k)+1(-2+3k)=0$

$\Rightarrow -8+4k-2+3k=0$

$\Rightarrow 7k=10$

$\therefore k=\dfrac{10}{7}$.

Ex04. Without actually expanding, evaluate the determinant : $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$.

Sol. Let $A=\begin{bmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{bmatrix}$ such that $\Delta=|A|$.

Note that $A'=\begin{bmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{bmatrix} = -A$.

Now $|A'|=|-A|$ $\{\because |kA|=k^n|A|$, where n is order of A and, $k\in R$

$\Rightarrow |A|=(-1)^3|A|$ $\{\because |A|=|A'|$

$\Rightarrow |A|=-|A|$

$\Rightarrow 2|A|=0 \quad \therefore |A|=0$.

☑ Since A is skew-symmetric matrix of order 3 (odd order) so, $\Delta=|A|=0$.

Ex05. If $\Delta=\begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{vmatrix}=-4$, then find the value of $\begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}$.

Sol. Given that $\Delta=\begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{vmatrix}=-4$.

Consider C_{ij} be the cofactor of element a_{ij}.

Then $C_{11}=a^3-1,\ C_{12}=0,\ C_{13}=a-a^4;\ C_{21}=0,\ C_{22}=a-a^4,\ C_{23}=a^3-1;\ C_{31}=a-a^4,$ $C_{32}=a^3-1,\ C_{33}=0$.

So, determinant formed by using the cofactors of Δ is $\begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}=\Delta_1$ say.

As we know that $\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}=\Delta^{3-1}=\Delta^2$.

Here we have used $|adj.A| = |A|^{n-1}$, where n is order of A; also $|A| = |A^T|$.

Hence $\Delta_1 = (-4)^2 = 16$.

Ex06. Prove that $\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix} = 6x^2(1-x^2)$.

Sol. LHS : Let $\Delta = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x) & x(x-1)(x-2) & x(x-1)(x+1) \end{vmatrix}$

$\Rightarrow \Delta = 1\left[x^2(x-1)^2(x+1) - x^2(x^2-1)(x-2)\right] - x\left[2x^2(x^2-1) - 3x^2(1-x^2)\right]$
$+(x+1)\left[2x^2(x-1)(x-2) - 3x^2(1-x)(x-1)\right]$

$\Rightarrow \Delta = \left[(x-1)^2(x^3+x^2) - (x^4-x^2)(x-2)\right] - x\left[2x^4-2x^2-3x^2+3x^4\right]$
$+(x+1)\left[2x^2(x^2-3x+2) + 3x^2(1-x)^2\right]$

$\Rightarrow \Delta = \left[(x^2-2x+1)(x^3+x^2) - x^5+2x^4+x^3-2x^2\right] - x\left[5x^4-5x^2\right]$
$+(x+1)\left[2x^4-6x^3+4x^2+3x^2(1-2x+x^2)\right]$

$\Rightarrow \Delta = \left[x^5+x^4-2x^4-2x^3+x^3+x^2-x^5+2x^4+x^3-2x^2\right] - 5x^5+5x^3$
$+(x+1)\left[2x^4-6x^3+4x^2+3x^2-6x^3+3x^4\right]$

$\Rightarrow \Delta = \left[x^4-x^2\right] - 5x^5+5x^3+(x+1)\left[5x^4-12x^3+7x^2\right]$

$\Rightarrow \Delta = x^4-x^2-5x^5+5x^3+\left[5x^5-12x^4+7x^3+5x^4-12x^3+7x^2\right]$

$\Rightarrow \Delta = -6x^4+6x^2$

$\Rightarrow \Delta = 6x^2(1-x^2) = \text{RHS}$.

EXERCISE 1.11

Q01. **(a)** Determine the value of the determinant: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$.

(b) Write the value of $\begin{vmatrix} \sin 20^\circ & \cos 20^\circ \\ -\sin 70^\circ & \cos 70^\circ \end{vmatrix}$.

(c) Find the value of $\begin{vmatrix} p & 0 & 0 \\ a & q & 0 \\ b & c & r \end{vmatrix}$.

(d) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number n, find the value of $\text{Det}(A^n)$.

Q02. Find the value of xy, if $\begin{vmatrix} 3x^3 & 8 \\ -4 & 4y^3 \end{vmatrix} = -4$.

Q03. **(a)** If $\begin{vmatrix} 3x & 1 \\ 5 & -x \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 7 & 2 \end{vmatrix}$, find the value (s) of x.

(b) If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

(c) If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x.

Q04. If $x \in R, 0 \le x \le \frac{\pi}{2}$, and $\begin{vmatrix} 2\sin x & -1 \\ 1 & \sin x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -4 & \sin x \end{vmatrix}$, then find the values of x.

Q05. If $A = [a_{ij}]$ is a 3×3 matrix and A_{ij} denotes the co-factors of the corresponding elements a_{ij}'s then, what is the value of $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$?

Q06. If $A = [a_{ij}]$ is a 3×3 matrix and M_{ij}'s denotes the minors of the corresponding elements a_{ij}'s then, write the expression for the value of $|A|$ by expanding $|A|$ by third column.

Q07. Find the minor of the element 8 in the following determinant : $\Delta = \begin{vmatrix} 2 & 4 & 7 \\ 3 & 6 & 8 \\ -2 & -3 & 1 \end{vmatrix}$.

Q08. **(a)** Find the equation of line joining the points (1, 2) and (3, 6) using determinants.

(b) Show that the points $(a, b+c)$, $(b, c+a)$ and $(c, a+b)$ are collinear.

(c) Find the value of x, if area of a Δ is 35 sq. units with the vertices $(x, 4), (2, -6)$ and $(5, 4)$.

Q09. **(a)** If ω is a complex cube root of unity, then find the value of : $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$.

(*Use* $\omega^3 = 1, 1 + \omega + \omega^2 = 0$ *; if needed*).

(b) Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$.

Q10. If A, B, C are angles of a triangle, then find the value of $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$.

Q11. Evaluate the determinants given below:

(a) $\begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$ **(b)** $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$ **(c)** $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

(d) $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ **(e)** $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix}$ **(f)** $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$

(g) $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

***Q12.** Evaluate the followings using properties of determinants:

(a) $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$ **(b)** $\begin{vmatrix} x+a & a & a \\ b & x+b & b \\ c & c & x+c \end{vmatrix}$ **(c)** $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$.

We are aware that **Properties of Determinants are Deleted. Still we advice the learners to go through some basic properties. It shall be helpful in 1-markers and in MCQ-based Entrance Exams (such as CUET, NDA, JEE etc.).*

ⓘ In CBSE **2026** Exams, a **5-Marker question** was based on the Properties of Determinants. You may check this same question in **SELECTED HOTS QUESTIONS** given at the end in this book.

EXERCISE 1.12

Q01. Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \le \theta \le 2\pi$.

Let determinant of matrix A is $|A| = \Delta$. Then prove that $2 \le \Delta \le 4$.

Q02. Evaluate the followings:

(a) $\begin{vmatrix} 0 & -b & c \\ b & 0 & a \\ -c & -a & 0 \end{vmatrix}$ **(b)** $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$

(c) $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$ **(d)** $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

Q03. If $\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$, then prove that $\Delta + \Delta_1 = 0$.

Q04. Prove that $\begin{vmatrix} x & \sin\delta & \cos\delta \\ -\sin\delta & -x & 1 \\ \cos\delta & 1 & x \end{vmatrix}$ is independent of δ.

Q05. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, write the value of x.

Q06. Find the values of x and y, if $\begin{vmatrix} x & -3i & 1 \\ y & 1 & i \\ 0 & 2i & -i \end{vmatrix} = 6 + 11i$.

Q07. Without expanding the determinant at any stage, prove that $\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix} = 0$.

Q08. If [.] denotes the greatest integer function, and $-1 \le x < 0$, $0 \le y < 1$, $1 \le z < 2$, then find value of the determinant: $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$.

Q09. Evaluate the determinant: $\begin{vmatrix} \log 3x & \log 3y & \log 3z \\ \log 2x & \log 2y & \log 2z \\ \log x & \log y & \log z \end{vmatrix}$.

EXERCISE 1.13

Q01. Prove the followings :

(a) $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 3 & 4+3p & 2+4p+3q \\ 4 & 7+4p & 2+7p+4q \end{vmatrix} = 1$

(b) $\begin{vmatrix} 2ab & a^2 & b^2 \\ a^2 & b^2 & 2ab \\ b^2 & 2ab & a^2 \end{vmatrix} = -\left(a^3+b^3\right)^2$

(c) $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} = 0$

(d) $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

(e) $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

(f) $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

(g) $\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$

(h) $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$

(i) $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc-a^3-b^3-c^3$

(j) $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

(k) $\begin{vmatrix} x^2+2x & 2x+1 & 1 \\ 2x+1 & x+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = \left(x-1\right)^3$

(l) $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

(m) $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2 = (2abc)^2$

(n) $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

(o) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$

(p) $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$ **(q)** $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$

EXERCISE 1.14

Q01. Solve for x : $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$.

Q02. Solve : $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0,\ a \neq 0$.

Q03. Write the value of the determinant : $\Delta = \begin{vmatrix} 1 & ab & \frac{1}{a}+\frac{1}{b} \\ 1 & bc & \frac{1}{b}+\frac{1}{c} \\ 1 & ca & \frac{1}{c}+\frac{1}{a} \end{vmatrix}$.

Hence, find $(2026)^{\Delta}$.

Q04. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then find the value of $f(2x)-f(x)$. Hence, find $f'(x)$ and write the value of $f'(x)$ when $x=0$.

✪ Adjoint of a square matrix

01. Def. Let $A=[a_{ij}]$ be a square matrix. Also assume $B=[A_{ij}]$ where A_{ij} is the cofactor of the elements a_{ij} in matrix A. Then the transpose B^T of matrix B is called the ***adjoint of matrix A*** and it is denoted by "***adj.*A**".

> **To find the adjoint of a 2×2 matrix :** Follow this, $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\therefore adj.A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

For example, consider a square matrix of order 3 as $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 2 & 0 & 5 \end{bmatrix}$ then, in order to find the adjoint of matrix A, we find a matrix B (formed by the cofactors of elements of det. corresponding to matrix A as mentioned above in the definition) *i.e.*, $B=\begin{bmatrix} 15 & -2 & -6 \\ -10 & -1 & 4 \\ -1 & 2 & -1 \end{bmatrix}$. Hence, $adj.A = B^T = \begin{bmatrix} 15 & -10 & -1 \\ -2 & -1 & 2 \\ -6 & 4 & -1 \end{bmatrix}$.

02. Singular matrix & Non-singular matrix :

(a) Singular matrix :

A square matrix A is said to be singular matrix if $|A|=0$ i.e., its **determinant is *zero***.

e.g. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 12 \\ 1 & 1 & 3 \end{bmatrix}, \begin{bmatrix} -3 & 4 \\ 3 & -4 \end{bmatrix}$.

(b) Non-singular matrix :

A square matrix A is said to be non-singular matrix if $|A| \neq 0$.

e.g. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 4 \\ -1 & 1 \end{bmatrix}$.

✪ *A square matrix A is **invertible** if and only if A is **non-singular matrix**.*

03. Algorithm to find A^{-1} by Determinant method :

STEP 1 Find $|A|$.

STEP 2 If $|A|=0$, then write **"A is a singular matrix and hence not invertible"**.
Else, write **"A is a non-singular matrix and hence invertible"**.

STEP 3 Calculate the cofactors of elements of matrix A.

STEP 4 Write the matrix of cofactors of elements of A and, then obtain its transpose to get *adj*.A (*i.e.*, *adjoint* A).

STEP 5 Find the inverse of A, by using the relation $A^{-1} = \frac{1}{|A|} \, adj.A$.

04. Properties associated with various operations of Matrices & Determinants :

(a) $AB = I = BA \Rightarrow A^{-1} = B$ and $B^{-1} = A$

(b) $AA^{-1} = I$ or, $A^{-1}A = I$, $A^{-1}I = A^{-1}$

(c) $(AB)^{-1} = B^{-1}A^{-1}$

(d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

(e) $(A^{-1})^{-1} = A$

(f) $(A^T)^{-1} = (A^{-1})^T$

(g) $A(adj.A) = (adj.A)A = |A|\, I$

(h) $adj.(AB) = (adj.B)(adj.A)$

(i) $adj.(A^T) = (adj.A)^T$

(j) $(adj.A)^{-1} = (adj.A^{-1})$

(k) $|adj.A| = |A|^{n-1}$ where n is order of A (if $|A| \neq 0$ i.e., A is a non-singular matrix)

(l) $|AB| = |A||B|$

(m) $|A \, . \, adj.A| = |A|^n$, where n is order of A

(n) $|A^{-1}| = \frac{1}{|A|}$, iff matrix A is invertible

(o) $|A| = |A^T|$

(p) $|kA| = k^n |A|$ where n is order of square matrix A and k is any scalar.

(q) If A is a non-singular matrix of order n, then $adj.(adj.A) = |A|^{n-2} A$.

(r) If A is a non-singular matrix of order n, then $|adj.(adj.A)| = |adj.A|^{n-1} = |A|^{(n-1)(n-1)} = |A|^{(n-1)^2}$.

(s) If A is a matrix of order n, then $adj.(kA) = k^{n-1}(adjA)$, $k \in R$.

(t) If A is skew-symmetric matrix of odd order, then $|A| = 0$.

(u) For a square matrix A, $|A^n| = |A|^n$, where $n \in N$.

WORKED OUT ILLUSTRATIVE EXAMPLES

Ex01. (a) If $|A| = 3$ and $A^{-1} = \begin{bmatrix} 3 & -1 \\ -5/3 & 2/3 \end{bmatrix}$, then write the adj.A.

(b) If for any 2×2 square matrix A, $A(\text{adj}.A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of |A|.

(c) Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$. Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

(d) Find $|AB|$, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

(e) If $A = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3| = 125$, then find the value of 'p'.

Sol. (a) $\because A^{-1} = \frac{1}{|A|}\text{adj}.A$

$\therefore \text{adj}.A = |A|\ A^{-1} = 3\begin{bmatrix} 3 & -1 \\ -5/3 & 2/3 \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$.

(b) As $A(\text{adj}.A) = |A|\ I = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 8\ I$.

On comparing, we get : $|A| = 8$.

(c) Let $A = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} -3 & 2 \\ 5 & -3 \end{vmatrix} = 9 - 10 = -1$, $\text{adj}.A = \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$

$\therefore A^{-1} = \frac{\text{adj}.A}{|A|} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ ...(i)

Now $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$\Rightarrow P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$ (On post-multiplication with A^{-1}

By (i), we get : $P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

$\therefore P = \begin{bmatrix} 3+10 & 2+6 \\ 6-5 & 4-3 \end{bmatrix} = \begin{bmatrix} 13 & 8 \\ 1 & 1 \end{bmatrix}$.

(d) $|AB| = |A||B| = \begin{vmatrix} 0 & -1 \\ 0 & 2 \end{vmatrix}\begin{vmatrix} 3 & 5 \\ 0 & 0 \end{vmatrix} = 0 \times 0 = 0$.

(e) $\because |A^3| = |A|^3 = 125$

$\Rightarrow |A| = \begin{vmatrix} p & 2 \\ 2 & p \end{vmatrix} = 5$

$\Rightarrow p^2 - 4 = 5$

$\therefore p = \pm 3$.

Ex02. In the interval $\frac{\pi}{2} < x < \pi$, find the value of x for which $\begin{pmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{pmatrix}$ is singular.

Sol. As $\begin{pmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{pmatrix}$ is singular matrix so, $\begin{vmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{vmatrix} = 0$

$\Rightarrow 4\sin^2 x - 3 = 0$

$\Rightarrow \sin^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$ $\left(\because \frac{\pi}{2} < x < \pi \ \therefore \sin x \neq -\frac{\sqrt{3}}{2} \right.$

$\therefore x = \frac{2\pi}{3}$.

Ex03. (a) If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$.

(b) If A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 3$, then find the value of $|2AB|$.

(c) If A and B are square matrices, each of order 2 such that $|A| = 3$ and $|B| = -2$, then write the value of $|3AB|$.

(d) If A and B are invertible matrices of order 3, $|A| = 2$ and $|(AB)^{-1}| = -\frac{1}{6}$, find $|B|$.

(e) If A and B are square matrices of the same order 3, such that $|A| = 2$ and $AB = 2I$, write the value of $|B|$.

(f) If A is a square matrix satisfying $A'A = I$, write the value of $|A|$.

(g) If A is a square matrix of order 3, with $|A| = 9$, then write the value of $|2\ \text{adj}.A|$.

Sol. **(a)** As $|A^{-1}| = |A|^{-1}$ $\therefore k = -1$.

(b) $|2AB| = 2^3|AB| = 8|A||B| = 8(-1)(3) = -24$.

#We have used $|kA| = k^n|A|$, where n is order of A. Also $|AB| = |A||B|$.

(c) $|3AB| = 3^2|AB| = 9|A||B| = 9 \times 3 \times (-2) = -54$.

(d) As $|P^{-1}| = \frac{1}{|P|}$

$\therefore |(AB)^{-1}| = \frac{1}{|AB|} = \frac{1}{|A||B|} = \frac{1}{2 \times |B|}$

Also since $|(AB)^{-1}| = -\frac{1}{6}$ so, $\frac{1}{2 \times |B|} = -\frac{1}{6}$.

$\therefore |B| = -3$.

(e) As $AB = 2I$ $\Rightarrow |AB| = |2I|$

$\Rightarrow |A||B| = 2^3|I|$

$\Rightarrow 2|B| = 8\times 1$

$\therefore |B| = 4$.

Note that the order of A and B is 3 and $AB = 2I$ so, I is also of order 3.

(f) $A'A = I \quad \Rightarrow |A'A| = |I| \quad \Rightarrow |A'||A| = 1$

$\Rightarrow |A|^2 = 1$

$\therefore |A| = \pm 1$.

(g) $|2\,\text{adj}.A| = 2^3|\text{adj}.A| = 8\times|A|^{3-1} = 8\times 9^2 = 648$.

Ex04. If A is a skew-symmetric matrix of order 3, then prove that $\det A = 0$.

Sol. As A is skew-symmetric of order 3.

So, $A = -A^T$.

Now $|A| = |-A^T| = (-1)^3|A^T| = -|A|$ $\quad \{\because |kA| = k^n|A|$, where n is order of A; $|A| = |A^T|$

$\Rightarrow |A| + |A| = 0$

$\therefore |A| = 0$ or, $\det A = 0$.

Can you prove that **Det. (A) is always 0, if A is a skew-symmetric matrix of odd order**?

Ex05. Find the value of m, if $\begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$.

Sol. We have $\begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$ $\quad \{\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

By $C_2 \to C_2 + C_1$, $\begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 + {}^{10}C_4 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 + {}^{11}C_6 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 + {}^{12}C_8 & {}^{13}C_{m+4} \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} {}^{10}C_4 & {}^{11}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{12}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{13}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$

Since $\Delta = 0$ so, C_2 and C_3 must be identical so, $m = 5$.

Ex06. Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$.

Sol. Here $|A| = \begin{vmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 5(-1) - 0 + 4(1) = -1$.

Consider A_{ij} be the cofactors of element a_{ij} of matrix A.

$A_{11} = -1$, $\quad A_{21} = 8$, $\quad A_{31} = -12$

$A_{12} = 0$, $\quad A_{22} = 1$, $\quad A_{32} = -2$

$A_{13} = 1$, $\quad A_{23} = -10$, $\quad A_{33} = 15$

So, $A^{-1} = \frac{1}{|A|}\text{adj.}A = \frac{1}{-1}\begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix} = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$

Hence, $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$

$\therefore (AB)^{-1} = \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$.

Ex07. Verify : $A(\text{adj}A) = (\text{adj}A)A = |A|\, I$ for matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$.

Sol. For the given matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = 11$

$\therefore |A| I = 11\, I \quad \ldots(i)$

Now $\text{adj.}A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$.

Therefore, $A(\text{adj.}A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = 11\, I \quad \ldots(ii)$

Also, $(\text{adj.}A)A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = 11\, I \quad \ldots(iii)$

By (i), (ii) and (iii), we see that, $A(\text{adj}A) = (\text{adj}A)A = |A|\, I$.

Ex08. Find the matrix A, if $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$.

Sol. Given $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1}\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$

$\Rightarrow A = \left(\frac{1}{4-3}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\right)\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}\left(\frac{1}{9-10}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}\right)$

$\Rightarrow A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

$$\Rightarrow A=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}\begin{bmatrix}14 & 8\\ 4 & 3\end{bmatrix}$$

$$\therefore A=\begin{bmatrix}24 & 13\\ -34 & -18\end{bmatrix}.$$

Ex09. If $A=\begin{bmatrix}-1 & a & 2\\ 1 & 2 & x\\ 3 & 1 & 1\end{bmatrix}$ **and** $A^{-1}=\begin{bmatrix}1 & -1 & 1\\ -8 & 7 & -5\\ b & y & 3\end{bmatrix}$**, find the value of** $(a+x)-(b+y)$**.**

Sol. Since $AA^{-1}=I$ so, $\begin{bmatrix}-1 & a & 2\\ 1 & 2 & x\\ 3 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ -8 & 7 & -5\\ b & y & 3\end{bmatrix}=I$

$$\Rightarrow\begin{bmatrix}-1-8a+2b & 1+7a+2y & 5-5a\\ -15+bx & 13+xy & -9+3x\\ -5+b & 4+y & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By equality of matrices, we get

$5-5a=0\Rightarrow a=1;\ b-5=0\Rightarrow b=5;\ 4+y=0\Rightarrow y=-4;\ -9+3x=0\Rightarrow x=3$

Hence, $(a+x)-(b+y)=(1+3)-(5-4)=4-1=3$.

Ex10. If $A=\begin{bmatrix}1 & \cot x\\ -\cot x & 1\end{bmatrix}$**, show that** $A'A^{-1}=\begin{bmatrix}-\cos 2x & -\sin 2x\\ \sin 2x & -\cos 2x\end{bmatrix}$**.**

Sol. For $A=\begin{bmatrix}1 & \cot x\\ -\cot x & 1\end{bmatrix}$, $A'=\begin{bmatrix}1 & -\cot x\\ \cot x & 1\end{bmatrix}$ and $A^{-1}=\dfrac{1}{1+\cot^2 x}\begin{bmatrix}1 & -\cot x\\ \cot x & 1\end{bmatrix}$

LHS : $A'A^{-1}=\begin{bmatrix}1 & -\cot x\\ \cot x & 1\end{bmatrix}\dfrac{1}{1+\cot^2 x}\begin{bmatrix}1 & -\cot x\\ \cot x & 1\end{bmatrix}$

$$=\frac{1}{1+\cot^2 x}\begin{bmatrix}1-\cot^2 x & -2\cot x\\ 2\cot x & 1-\cot^2 x\end{bmatrix}$$

$$=\begin{bmatrix}\dfrac{1-\cot^2 x}{1+\cot^2 x} & -\dfrac{2\cot x}{1+\cot^2 x}\\ \dfrac{2\cot x}{1+\cot^2 x} & \dfrac{1-\cot^2 x}{1+\cot^2 x}\end{bmatrix}=\begin{bmatrix}-\dfrac{1-\tan^2 x}{1+\tan^2 x} & -\dfrac{2\tan x}{1+\tan^2 x}\\ \dfrac{2\tan x}{1+\tan^2 x} & -\dfrac{1-\tan^2 x}{1+\tan^2 x}\end{bmatrix}$$

$$=\begin{bmatrix}-\cos 2x & -\sin 2x\\ \sin 2x & -\cos 2x\end{bmatrix}$$

$= \text{RHS}.$

Ex11. If $A=\begin{bmatrix}5 & 0 & 4\\ 2 & 3 & 2\\ 1 & 2 & 1\end{bmatrix}$ **and** $B^{-1}=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$**, find** $(AB)^{-1}$**. Also, find** $\left|(AB)^{-1}\right|$**.**

Sol. For $A=\begin{bmatrix}5 & 0 & 4\\ 2 & 3 & 2\\ 1 & 2 & 1\end{bmatrix}$, $|A|=\begin{vmatrix}5 & 0 & 4\\ 2 & 3 & 2\\ 1 & 2 & 1\end{vmatrix}=5(-1)-0+4(1)=-1$

Since $|A|\neq 0\ \therefore A^{-1}$ exists.

Also, $\text{adj.A} = \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$ $\Rightarrow A^{-1} = \dfrac{\text{adj.A}}{|A|} = \dfrac{1}{-1}\begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$

Now $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$

$\Rightarrow (AB)^{-1} = \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$

Also, $|(AB)^{-1}| = |B^{-1}A^{-1}| = |B^{-1}||A^{-1}| = [1(7)-3(1)+3(-1)][1(-5)-0+(-1)(-4)]$

$\Rightarrow |(AB)^{-1}| = (1)(-1) = -1$.

EXERCISE 1.15

Q01. **(a)** Let $A = 2B$, where A and B both are square matrices of 3^{rd} order and $|B| = 5$. Find $|A|$.

(b) If A is a square matrix such that $A(adjA) = 5I$, then determine the value of $|A|$.

(c) If A is a square matrix of order 3 such that $|A| = 5$, then determine the value of $|adjA|$.

(d) If A is a square matrix of order 3 such that $|adjA| = 64$, then find $|A|$.

(e) If A is a non-singular square matrix such that $|A| = 10$, then determine the value of $|A^{-1}|$.

(f) If A is a square matrix of order 3 such that $A(adjA) = 5I$, find $|adjA|$.

(g) Assume that A is a square matrix of order 3, then what will be the value of $|kA|$.

(h) If $|adj.A| = 36$, then find $|3A^{-1}|$ if A is a square matrix of order 3.

(i) If A is a non-singular square matrix of order 3 then, determine the value of $|adj.A|$.

(j) If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k\,|A|$, then write the value of k.

(k) If A is a square matrix such that $|A| = 5$, write the value of $|AA^T|$.

(l) A and B are square matrices of order 3 each, $|A| = 2$ and $|B| = 3$. Find $|3AB|$.

Q02. **(a)** For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

(b) For what value of x, the matrix $\begin{pmatrix} 2-x & 3 \\ -5 & 1 \end{pmatrix}$ is non-invertible?

(c) For what value of x, the matrix $\begin{pmatrix} 7-2x & x+5 \\ 3 & 7 \end{pmatrix}$ is singular?

(d) If $0 < x < \pi$, and the matrix $\begin{pmatrix} 3 & 2\sin x \\ 2\sin x & 1 \end{pmatrix}$ is singular, write the value(s) of x.

Q03. Prove that $(A^{-1})' = (A')^{-1}$, where A is an invertible matrix.

Q04. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then write A^{-1}.

Q05. If x, y, z are all non-zero real numbers, then find $\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}^{-1}$.

Q06. Find A . (adj.A) without finding adj.A, if $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$.

Q07. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$, then find the value of $\left|A^{2009} - 5A^{2008}\right|$.

Q08. If $A_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$ and $|A_1| + |A_2| + ... + |A_{2020}| = k^2$, find k, $(k > 0)$.

EXERCISE 1.16

Q01. **(a)** If $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix}$ and $AB - CD = O$, then find the matrix D.

(b) Let $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$. Find a matrix D, such that $CD - AB = O$.

Q02. Find the matrix A, satisfying the matrix equation : $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = I$.

Q03. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Q04. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -1 & 0 \\ 3 & 4 \end{bmatrix}$, then find $(AB)^{-1}$.

Q05. **(a)** Let $A = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$. Then verify $A(adjA) = (adjA)A = |A|\,I$.

(b) If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that $A(adj.A) = |A|\,I$. Also find A^{-1}.

Q06. Show that : $\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

Q07. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then find $(A')^{-1}$.

Q08. Find matrix A, in the followings :

(a) $A\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$ **(b)** $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6\,I_2$

(c) $\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix} A = \begin{bmatrix} -1 & 2 \\ 0 & 4 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

Q09. Find the inverse of matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ and, hence show that $A^{-1}.\, A = I$.

Q10. If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$, find adj.A and verify that $A(adj.A) = (adj.A)A = |A| I_3$.

✪ APPLICATION OF MATRICES & DETERMINANTS

INTRODUCTION

The study of matrices and determinants can be utilized to verify the Consistency of system of linear equations. We'll use inverse of matrix to solve system of linear equations. We'll also be discussing real life based problems and see the usage of matrices and determinants to solve such problems.

IMPORTANT TERMS, DEFINITIONS & RESULTS

01. Solutions of System of Linear equations :

(a) Consistent and Inconsistent system :

A system of equations is said to be consistent if it has *one or more* solutions otherwise it is said to be an inconsistent system. In other words an inconsistent system of equations has *no solution*.

(b) Homogeneous and Non-homogeneous system :

A system of equations $AX = B$ is said to be a homogeneous system if $B = O$.
Otherwise it is called a non-homogeneous system of equations.

02. Solving of system of equations by Matrix method (Inverse Matrix Method) :

Consider the following system of equations,

$$a_1x + b_1y + c_1z = d_1,$$
$$a_2x + b_2y + c_2z = d_2,$$
$$a_3x + b_3y + c_3z = d_3$$

STEP 1 Assume $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

STEP 2 Find $|A|$.

Now there may be following situations:

(a) $|A| \neq 0 \Rightarrow A^{-1}$ exists.

It implies that the given system of equations is *consistent* and therefore, the system has ***unique solution***.

In that case, write $AX = B$.

Pre-multiplying by A^{-1} both sides, we get : $A^{-1}AX = A^{-1}B$

$\Rightarrow I\,X = A^{-1}B$

$\therefore\ X = A^{-1}B$ $\left[\text{where } A^{-1} = \frac{1}{|A|}(adj.A)\right.$

Then by using the ***definition of equality of matrices***, we can get the values of x, y and z.

(b) $|A| = 0$ implies A^{-1} doesn't exist.

It implies that the given system of equations may be *consistent* or *inconsistent*.
In order to check proceed as follow:

- Find (*adj*.A) B.

Now we may have either $(adj.A)B \neq O$ or $(adj.A)B = O$.

(i) If $(adj.A)B = O$, then the given system may be *consistent* or *inconsistent*.
To check, put $z = k$ in the given equations and proceed in the same manner in the new *two variables* system of equations assuming $d_i - c_i k,\ 1 \leq i \leq 3$ as constant.

(ii) If $(adj.A)B \neq O$, then the given system is *inconsistent* with *no solutions*.

A Note about Homogeneous System of Equations :
The system of homogeneous equations $a_1x + b_1y + c_1z = 0,\ a_2x + b_2y + c_2z = 0,$ $a_3x + b_3y + c_3z = 0$ is always consistent.

- If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$, then system of equations has the unique solution $x = y = z = 0$ (known as **trivial solution**). But if $\Delta = 0$, then this system of equations has an infinite number of solution. Hence for non-trivial solution, we can say $\Delta = 0$.

WORKED OUT ILLUSTRATIVE EXAMPLES

Ex01. For what values of k, the system of linear equations $x + y + z = 2,\ 2x + y - z = 3,\ 3x + 2y + kz = 4$ has a unique solution?

Sol. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{pmatrix}$, which is matrix formed by the coefficients of x, y and z in the given system of equations.

For unique solution $|A| \neq 0$ so, $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$

$\Rightarrow 1(k+2) - 1(2k+3) + 1(4-3) \neq 0$
$\Rightarrow k + 2 - 2k - 3 + 1 \neq 0$
$\therefore k \neq 0$.

Ex02. (a) For what values of k and m, the system of linear equations $2x + ky + 6z = 8,\ x + 2y + mz = 5,\ x + y + 3z = 4$ has a unique solution?

(b) For what values of k, the system of linear equations $2x + ky + 6z = 8,\ x + 2y + z = 5,\ x + y + 3z = 4$ has infinitely many solutions?

(c) For what values of k, the system of linear equations $2x + ky + 6z = 8,\ x + 2y + z = 5,\ x + y + 3z = 4$ has no solution?

Sol. **(a)** Let $A=\begin{pmatrix}2 & k & 6\\ 1 & 2 & m\\ 1 & 1 & 3\end{pmatrix}$, which is matrix formed by the coefficients of x, y and z in the given system of equations.

For unique solution, $|A|\neq 0$ so, $\begin{vmatrix}2 & k & 6\\ 1 & 2 & m\\ 1 & 1 & 3\end{vmatrix}\neq 0$

$\Rightarrow 2(6-m)-k(3-m)+6(-1)\neq 0$

$\Rightarrow 12-2m-3k+mk-6\neq 0$

$\Rightarrow 6-2m-3k+mk\neq 0$

$\Rightarrow (k-2)(m-3)\neq 0$

. Clearly, $k-2\neq 0,\ m-3\neq 0$

$\therefore k\neq 2,\ m\neq 3$.

(b) Let $A=\begin{pmatrix}2 & k & 6\\ 1 & 2 & 1\\ 1 & 1 & 3\end{pmatrix}$, which is matrix formed by the coefficients of x, y and z in the given system of equations.

Also, let $B=\begin{pmatrix}8\\ 5\\ 4\end{pmatrix}$, which is matrix formed by the constants in the given system of equations.

Let A_{ij} be the cofactor of element a_{ij} of matrix A.

$C_{11}=5,\ C_{12}=-2,\ C_{13}=-1,$

$C_{21}=6-3k,\ C_{22}=0,\ C_{23}=k-2,$

$C_{31}=k-12,\ C_{32}=4,\ C_{33}=4-k$

$\therefore \text{adj}.A=\begin{pmatrix}5 & 6-3k & k-12\\ -2 & 0 & 4\\ -1 & k-2 & 4-k\end{pmatrix}$

Now $(\text{adj}.A).B=\begin{pmatrix}5 & 6-3k & k-12\\ -2 & 0 & 4\\ -1 & k-2 & 4-k\end{pmatrix}\begin{pmatrix}8\\ 5\\ 4\end{pmatrix}=\begin{pmatrix}22-11k\\ 0\\ k-2\end{pmatrix}$

For infinitely many solutions, we must have $(\text{adj}.A).B=O$

$\Rightarrow\begin{pmatrix}22-11k\\ 0\\ k-2\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$

By equality of matrices, $22-11k=0,\ k-2=0$

$\therefore k=2$.

Alternatively, let $\Delta=\begin{vmatrix}2 & k & 6\\ 1 & 2 & 1\\ 1 & 1 & 3\end{vmatrix}=-2(k-2)$,

$\Delta_1=\begin{vmatrix}8 & k & 6\\ 5 & 2 & 1\\ 4 & 1 & 3\end{vmatrix}=8(6-1)-k(15-4)+6(-3)$

$\Rightarrow \Delta_1 = -11(k-2)$,

$$\Delta_2 = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & 1 \\ 1 & 4 & 3 \end{vmatrix} = 2(15-4) - 8(3-1) + 6(-1)$$

$\Rightarrow \Delta_2 = 0$,

$$\Delta_3 = \begin{vmatrix} 2 & k & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{vmatrix} = 2(3) - k(-1) + 8(-1)$$

$\Rightarrow \Delta_3 = k - 2$.

For infinitely many solutions, we must have $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

So, $\Delta = -2(k-2) = 0$ $\quad \therefore k = 2$,

$\Delta_1 = -11(k-2) = 0$ $\quad \therefore k = 2$,

$\Delta_3 = k - 2 = 0$ $\quad \therefore k = 2$.

Hence $k = 2$.

(c) For no solution, we must have $(adj.A).B \neq O$

$$\Rightarrow \begin{pmatrix} 22-11k \\ 0 \\ k-2 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ (Taking value of (adj.A) B from (b)

$22 - 11k \neq 0,\ k - 2 \neq 0$

$\Rightarrow k \neq 2$.

Alternatively, for no solution, we must have $\Delta = 0$ or, any One of $\Delta_1, \Delta_2, \Delta_3$ must be nonzero.

So, $\Delta = -2(k-2) = 0$ $\quad \therefore k = 2$,

Now note that we already have $\Delta_2 = 0$.

Then, either $\Delta_1 \neq 0$ or, $\Delta_3 \neq 0$.

That is, $\Delta_1 = -11(k-2) \neq 0$ $\quad \therefore k \neq 2$,

and, $\Delta_3 = k - 2 \neq 0$ $\quad \therefore k \neq 2$

Hence $k \neq 2$.

Note that in **(b)** and **(c)**, the equations are same.

Ex03. Solve the following system of equations using matrix method :
$x + 2y + z = 7,\ x + 3z = 11,\ 2x - 3y = 1$.

Sol. The given system of equations is :

$$x + 2y + z = 7,$$
$$x + 3z = 11,$$
$$2x - 3y = 1$$

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$.

Now, $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix} = 1(0+9) - 2(0-6) + 1(-3-0) = 18 \neq 0$.

So, A^{-1} exists.

Let A_{ij} be the cofactors of elements a_{ij} in $A=[a_{ij}]$. Then, we have :

$A_{11}=9,\quad A_{21}=-3,\quad A_{31}=6$

$A_{12}=6,\quad A_{22}=-2,\quad A_{32}=-2$

$A_{13}=-3,\quad A_{23}=7,\quad A_{33}=-2$

$$\therefore\ \text{adj}A=\begin{bmatrix}9&-3&6\\6&-2&-2\\-3&7&-2\end{bmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{|A|}(\text{adj}A)=\frac{1}{18}\begin{bmatrix}9&-3&6\\6&-2&-2\\-3&7&-2\end{bmatrix}$$

Now as $AX=B$

$\Rightarrow A^{-1}AX=A^{-1}B$ (Pre-multiplying by A^{-1} both the sides

$\Rightarrow IX=A^{-1}B$

$\Rightarrow X=A^{-1}B$

$$\text{So, } X=\frac{1}{18}\begin{bmatrix}9&-3&6\\6&-2&-2\\-3&7&-2\end{bmatrix}\begin{bmatrix}7\\11\\1\end{bmatrix}$$

$$\Rightarrow X=\frac{1}{18}\begin{bmatrix}63-33+6\\42-22-2\\-21+77-2\end{bmatrix}=\frac{1}{18}\begin{bmatrix}36\\18\\54\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}$$

Hence by equality of matrices, we get : $x=2, y=1$ and $z=3$.

Ex04. If $A=\begin{pmatrix}2&3&1\\1&2&2\\-3&1&-1\end{pmatrix}$, find A^{-1} and hence solve the system of equations $2x+y-3z=13,\ 3x+2y+z=4,\ x+2y-z=8$.

Sol. For $A=\begin{pmatrix}2&3&1\\1&2&2\\-3&1&-1\end{pmatrix}$, $|A|=\begin{vmatrix}2&3&1\\1&2&2\\-3&1&-1\end{vmatrix}=-16$.

Clearly A^{-1} exists as $|A|\neq 0$.

Since $\text{adj}.A=\begin{pmatrix}-4&4&4\\-5&1&-3\\7&-11&1\end{pmatrix}$ $\quad\therefore A^{-1}=-\frac{1}{16}\begin{pmatrix}-4&4&4\\-5&1&-3\\7&-11&1\end{pmatrix}$

Now $2x+y-3z=13,\ 3x+2y+z=4,\ x+2y-z=8$

By using matrix method, $A'X=B$ where $A=\begin{pmatrix}2&3&1\\1&2&2\\-3&1&-1\end{pmatrix}$, $X=\begin{pmatrix}x\\y\\z\end{pmatrix}$, $B=\begin{pmatrix}13\\4\\8\end{pmatrix}$

Now $X=(A')^{-1}B=(A^{-1})'B$

$$\Rightarrow X = -\frac{1}{16}\begin{pmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{pmatrix}\begin{pmatrix} 13 \\ 4 \\ 8 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$$

By using equality of matrices, we get : $x = 1,\ y = 2,\ z = -3$.

Ex05. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations $x - y = 3,\ 2x + 3y + 4z = 17$ and $y + 2z = 7$.

Sol. Here $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}.$$

That is, $AB = 6I$ ...(i)

Consider the given systems of equations : $x - y = 3,\ 2x + 3y + 4z = 17$ and $y + 2z = 7$

These equations can be expressed as : $PX = D$ where $P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

Therefore, $X = P^{-1}D = (A)^{-1}D = (A^{-1})D = \frac{1}{6}BD$

By (i), $AB = 6I \Rightarrow A^{-1} = \frac{1}{6}B$
$\because P = A \quad \therefore P^{-1} = A^{-1} = \frac{1}{6}B$

$$\text{So, } X = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} \qquad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get :
$x = 2,\ y = -1,\ z = 4$.

Ex06. Determine the product $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ **and use it to solve the system of equations :** $\mathbf{x-y+z=4,\ x-2y-2z=9,\ 2x+y+3z=1}$**.**

Sol. Let $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$, $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$

$$\therefore AB=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}=\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}$$

$\Rightarrow AB=8\,I$ ...(i)

Consider the given systems of equations : $x-y+z=4,\ x-2y-2z=9,\ 2x+y+3z=1$

These equations can be expressed as : $BX=D$ where $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $C=\begin{bmatrix}4\\ 9\\ 1\end{bmatrix}$

Therefore, $X=B^{-1}C=\frac{1}{8}AC$ $\quad\left[\text{By (i), } AB=8I\Rightarrow\left(\frac{1}{8}A\right)B=I\ \therefore B^{-1}=\frac{1}{8}A\right.$

So, $X=\frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\ 9\\ 1\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ -2\\ -1\end{bmatrix}$$

$\therefore$ By equality of matrices : $x=3, y=-2, z=-1$.

Ex07. Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ **to solve the system of equations** $\mathbf{x+3z=9,\ -x+2y-2z=4,\ 2x-3y+4z=-3}$**.**

Sol. Let $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$, $B=\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$

$$\therefore AB=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$$

$$\Rightarrow AB=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

That is, $AB=I$ ...(i)

Consider the given systems of equations : $x+3z=9,\ -x+2y-2z=4,\ 2x-3y+4z=-3$

These equations can be expressed as : $PX = D$ where $P = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, D = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$

Therefore, $X = P^{-1}D = (A^T)^{-1}D = (A^{-1})^T D$ $\quad \left[\begin{array}{l} \text{By (i), } AB = I \Rightarrow A^{-1} = B \\ \because P = A^T \quad \therefore P^{-1} = (A^{-1})^T = B^T \end{array}\right.$

So, $X = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$

$\therefore$ By equality of matrices : $x = 0, y = 5, z = 3$.

Ex08. **If $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$, find A^{-1}. Using A^{-1} solve the following system of equations :**

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4; \ x, y, z \neq 0.$$

Sol. Here $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$

So, $|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$\Rightarrow |A| = 1200 \neq 0$

$\therefore A^{-1}$ exists.

Consider A_{ij} be the cofactors of corresponding elements a_{ij} of matrix A.

$A_{11} = 75 \qquad A_{21} = 150 \qquad A_{31} = 75$

$A_{12} = 110 \qquad A_{22} = -100 \qquad A_{32} = 30$

$A_{13} = 72 \qquad A_{23} = 0 \qquad A_{33} = -24$

$\therefore \text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \qquad \Rightarrow A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

Now $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$

By using matrix method, $AX = B$ where $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}, B = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$

Note that, $AX = B$

On Pre-multiplication by A^{-1} we get : $A^{-1}AX = A^{-1}B$

$\therefore X = A^{-1}B$

$$\Rightarrow X = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/3 \\ 1/5 \end{bmatrix}$$

By using equality of matrices, we get : $\frac{1}{x} = \frac{1}{2}, \frac{1}{y} = -\frac{1}{3}, \frac{1}{z} = \frac{1}{5}$

$\therefore x = 2,\ y = -3,\ z = 5$.

Ex09. A school wants to award its students for the values Honesty, Regularity and Hard-work with a total cash award of ₹6000. Three times the award money for Hard-work added to that given for Honesty amounts to ₹11000. The award money given for Honesty and Hard-work together is double the one given for Regularity.
Represent the above situation algebraically and find the award money for each value, using matrix method.

Sol. Let the award money for the values of Honesty, Regularity and Hard-work be x, y and z (in ₹) respectively.

According to question, we get : $x + y + z = 6000,\ x + 3z = 11000,\ x - 2y + z = 0$.

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{vmatrix} = 6 \neq 0$$

$\therefore A^{-1}$ exists.

Consider C_{ij} be the cofactors of element a_{ij} in matrix A, we have

$C_{11} = 6,\quad C_{12} = 2,\quad C_{13} = -2$

$C_{21} = -3,\quad C_{22} = 0,\quad C_{23} = 3$

$C_{31} = 3,\quad C_{32} = -2,\quad C_{33} = -1$

$$\text{So, adj.}A = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}.$$

$$\Rightarrow A^{-1} = \frac{1}{|A|}(\text{adj.}A) = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}.$$

As $X = A^{-1}B$

$$\Rightarrow X = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

By equality of matrices, we get : $x = 500, y = 2000, z = 3500$.

Hence, award money given for the value of Honesty = ₹500, award money given for the value of Regularity = ₹2000 and, award money given for the value of Hard-work = ₹3500.

Ex10. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹10 more. However, if there were 16 children more, every one would have got ₹10 less.
Using matrix method, find the number of children and the amount distributed by Seema.

Sol. Let the number of children be x and the amount distributed by Seema for one child be ₹y.

So, $(x-8)(y+10) = xy$

$\Rightarrow 5x - 4y = 40 \ldots(i)$

and $(x+16)(y-10) = xy$

$\Rightarrow 5x - 8y = -80 \ldots(ii)$

To solve (i) and (ii), let $A = \begin{pmatrix} 5 & -4 \\ 5 & -8 \end{pmatrix}$, $B = \begin{pmatrix} 40 \\ -80 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$

$\because AX = B \quad \Rightarrow X = A^{-1}B$

Now $A^{-1} = \dfrac{1}{-40+20}\begin{pmatrix} -8 & 4 \\ -5 & 5 \end{pmatrix} = \dfrac{1}{20}\begin{pmatrix} 8 & -4 \\ 5 & -5 \end{pmatrix}$

$\therefore X = \dfrac{1}{20}\begin{pmatrix} 8 & -4 \\ 5 & -5 \end{pmatrix}\begin{pmatrix} 40 \\ -80 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 32 \\ 30 \end{pmatrix}$

Clearly $x = 32,\ y = 30$.

Hence the number of children = 32 and, the amount distributed by Seema = ₹30.

Ex11. In an electrical circuit, the voltage equations are given by:

$$\mathbf{5I_1 + 3I_2 + 2I_3 = 21,\ I_1 + I_2 + I_3 = 6,\ 3I_1 + 5I_2 + I_3 = 22}$$

where I_1, I_2, I_3 represent the currents in different branches.
Find the determinant of the coefficient matrix to check if the system has a unique solution.
Hence, solve the equations to obtain I_1, I_2 and I_3.

Sol. Let $A = \begin{pmatrix} 5 & 3 & 2 \\ 1 & 1 & 1 \\ 3 & 5 & 1 \end{pmatrix}$, $X = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$, $B = \begin{pmatrix} 21 \\ 6 \\ 22 \end{pmatrix}$ represent the system of given equations.

Since $|A| = \begin{vmatrix} 5 & 3 & 2 \\ 1 & 1 & 1 \\ 3 & 5 & 1 \end{vmatrix} = 5(-4) - 3(-2) + 2(2) = -10 \neq 0$.

Therefore, the system has a unique solution.

Now $AX = B \quad \Rightarrow X = A^{-1}B \ \ldots(i)$

Consider the cofactors A_{ij} of element a_{ij} for matrix A.

$A_{11} = -4,\ A_{12} = 2,\ A_{13} = 2;\ A_{21} = 7,\ A_{22} = -1,\ A_{23} = -16;\ A_{31} = 1,\ A_{32} = -3,\ A_{33} = 2$.

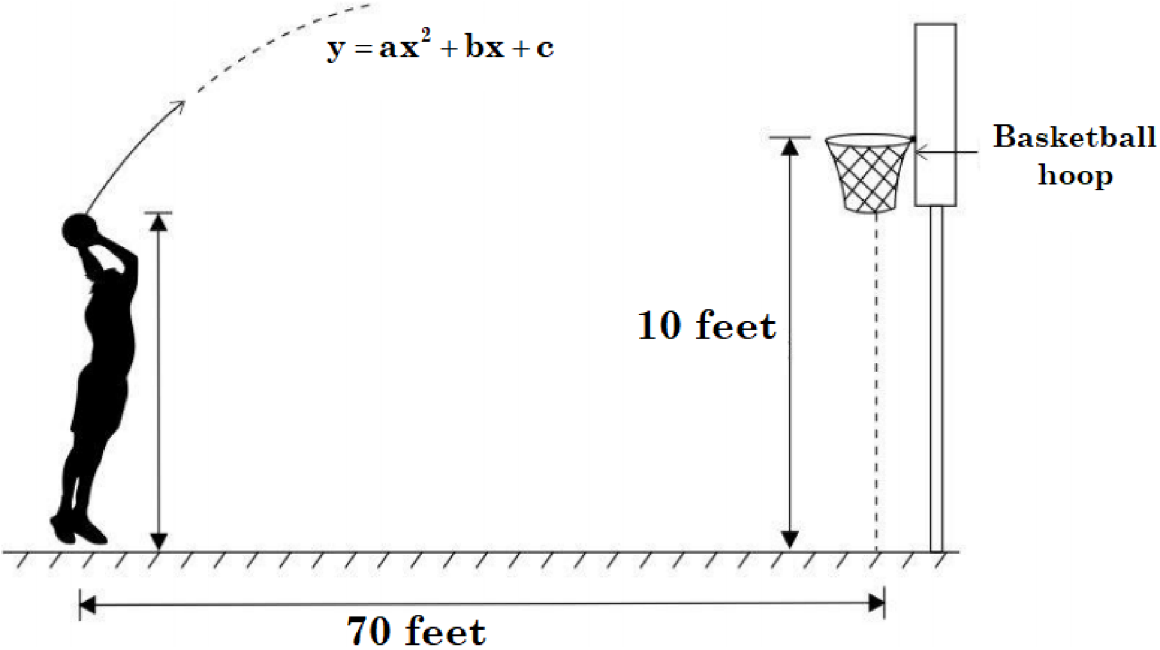
So, $\text{adj.}A = \begin{pmatrix} -4 & 7 & 1 \\ 2 & -1 & -3 \\ 2 & -16 & 2 \end{pmatrix}$ $\quad \therefore A^{-1} = \dfrac{\text{adj.}A}{|A|} = \dfrac{1}{-10}\begin{pmatrix} -4 & 7 & 1 \\ 2 & -1 & -3 \\ 2 & -16 & 2 \end{pmatrix}$

By (i), $X = \dfrac{1}{-10}\begin{pmatrix} -4 & 7 & 1 \\ 2 & -1 & -3 \\ 2 & -16 & 2 \end{pmatrix}\begin{pmatrix} 21 \\ 6 \\ 22 \end{pmatrix}$ $\quad \Rightarrow X = \dfrac{1}{-10}\begin{pmatrix} -20 \\ -30 \\ -10 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

By equality of matrices, we get : $I_1 = 2,\ I_2 = 3,\ I_3 = 1$.

Ex12. A class XII student, Abdul threw a basketball in the direction of the basketball hoop which traversed a parabolic path in a vertical plane as shown below.



(Note : The image is for representation purpose only.)

The equation of the path traversed by the ball is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane. The ball traversed through the points (10, 16), (20, 22) and (30, 25). The basketball hoop is at a horizontal distance of 70 feet from Abdul. The height of the basketball hoop is 10 feet from the floor to the top edge of the rim.

Did the ball successfully go through the hoop? Justify your answer.

Sol. Note that, the points $(10,16)$, $(20,22)$ and $(30,25)$ shall satisfy $y = ax^2 + bx + c$.

Therefore, the system of equations can be written as

$$100a + 10b + c = 16,$$
$$400a + 20b + c = 22,$$
$$900a + 30b + c = 25.$$

Expressing the system of equations in the form $AX = B$ as $\begin{pmatrix} 100 & 10 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 16 \\ 22 \\ 25 \end{pmatrix}$

Now $|A| = 1(18000-12000) - 1(3000-9000) + 1(2000-4000) = -2000$, A^{-1} exists as $|A| \neq 0$.

Also $\text{adj.}A = \begin{pmatrix} -10 & 20 & -10 \\ 500 & -800 & 300 \\ -6000 & 6000 & -2000 \end{pmatrix}$ and $A^{-1} = \dfrac{\text{adj.}A}{|A|} = \begin{pmatrix} \frac{1}{200} & \frac{-1}{100} & \frac{1}{200} \\ \frac{-1}{4} & \frac{2}{5} & \frac{-3}{20} \\ 3 & -3 & 1 \end{pmatrix}$.

Since $X = A^{-1}B$, then $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{1}{200} & \frac{-1}{100} & \frac{1}{200} \\ \frac{-1}{4} & \frac{2}{5} & \frac{-3}{20} \\ 3 & -3 & 1 \end{pmatrix} \times \begin{pmatrix} 16 \\ 22 \\ 25 \end{pmatrix} = \begin{pmatrix} \frac{-3}{200} \\ \frac{21}{20} \\ 7 \end{pmatrix}$

By equality of matrices, we get $a = -\dfrac{3}{200}$, $b = \dfrac{21}{20}$, $c = 7$.

Hence, the equation of the path traversed by the ball is $y = -\dfrac{3}{200}x^2 + \dfrac{21}{20}x + 7$.

Now when $x = 70$ feet, $y = 7$ feet.

So, the ball went by 7 feet above the floor that means 3 feet below the basketball hoop.
That means, the ball did not go through the hoop.

EXERCISE 1.17

Q01. Solve the given system of equations for x, y and z:

(a) $x + y = 4,\ 2x - 3y = 9$

(b) $-x + 2y + 3z = 3,\ 2x + 3y - 2z = 5,\ 3x + y + 4z = 11$

(c) $x + 2y + z = 7,\ x + 3z = 11,\ 2x - 3y = 1$

(d) $\dfrac{2}{x} + \dfrac{3}{y} + \dfrac{10}{z} = 4,\ \dfrac{4}{x} - \dfrac{6}{y} + \dfrac{5}{z} = 1,\ \dfrac{6}{x} + \dfrac{9}{y} - \dfrac{20}{z} = 2;\ \ x, y, z \neq 0$

(e) $3x + \dfrac{4}{y} + 7xz = 14,\ 2x - \dfrac{1}{y} + 3xz = 4,\ x + \dfrac{2}{y} - 3xz = 0$

Q02. If $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{bmatrix}$, then find A^{-1}.

Hence, solve the following system of equations:

$x + 2y + 3z = 8,$

$2x - y + 4z = 8,$

$5x + y - z = 16.$

Q03. Find the inverse of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 1 & 2 & 3 \\ 2 & 2 & -1 \end{bmatrix}$.

Hence solve the following system of equations:

$2x - z = 4,\ x + 2y + 3z = 0,\ 2x + 2y - z = 2.$

Q04. Let $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}4 & -6\\ -2 & 4\end{bmatrix}$. Then compute AB.
Hence, solve the given system of equations : $2x+y=4,\ 3x+2y=1$.

Q05. If $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$, $B=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$, then find the product AB.
Use this product to solve the system of equations : $2x-y+z=-1,\ -x+2y-z=4,$
$x-y+2z=-3$.

Q06. Use the product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the following system of equations:
$x-y+2z=1,\ 2y-3z=1,\ 3x-2y+4z=2$.

Q07. Let $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 2 & 0\\ 3 & 3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}-2 & 2 & -2\\ 2 & -4 & 2\\ 0 & -6 & 4\end{bmatrix}$, verify that $BA=-4I$, where I is a unit matrix.
Hence solve the given system of equations: $2y-2x-2z=0, 2x-4y+2z=2, -6y+4z=-8$.

Q08. Check if the following system of equations is consistent or inconsistent. If consistent, then solve:
(a) $2x+3y=5,\ 6x+9y=15$ **(b)** $2x+y-z=4,\ 3x+y-2z=6,\ x-z=2$.

EXERCISE 1.18

Q01. **(a)** A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of ₹21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for ₹60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹70. Using matrix method, find cost of each variety of pen.
(b) The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Find these three numbers by using matrix method.

Q02. Ishan wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.
Using matrices, find the dimensions of the plot.

Q03. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness and Leadership. The school P wants to award ₹x each, ₹y each and ₹z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of ₹2200. School Q wants to spend ₹3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as school P). If the total amount of award for one prize on each value is ₹1200, using matrices, find the award money for each value.

Q04. An amount of ₹10000 is put into three investments at the rate of interest 6%, 7% and 8% respectively. The total income is ₹716. Combined income from first two investments is ₹140 more than the income from third. Find the amount of each investment by matrix method.

Q05. There are three families. First family consists of 2 male members, 4 female members and 3 children. Second family consists of 3 male members, 3 female members and 2 children. Third family consists of 2 male members, 2 female members and 5 children. Male member earns ₹500 per day and spends ₹300 per day. Female member earns ₹400 per day and spends ₹250 per day, child member spends ₹40 per day. Find the money each family saves per day using matrices?

Q06. Mr. Nakul Saini has invested a part of his income in 10% (bond A) and another part of his income in 15% (bond B). His interest during a certain period is ₹4000. Had he invested 20% more in bond A and 10% more in bond B, his interest would have been increased by ₹500 for the same period. Then :
(i) Represent the above situation by a matrix equation and form linear equations using matrix multiplication.
(ii) Is it possible to solve the system of equations so obtained by matrices? If yes, solve it too.

Q07. In a parliament election in our country, a political party hired a public relation firm to promote its candidates in 3 ways – telephone, house calls and letters. The cost per contact (in paise) is given in matrix A as

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \begin{matrix} \text{Telephone} \\ \text{House Call} \\ \text{Letters} \end{matrix}$$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$\begin{matrix} \quad\text{Telephone} & \text{House Call} & \text{Letters} \end{matrix}$$
$$B = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{matrix} \text{City X} \\ \text{City Y} \end{matrix}$$

Find the total amount spent by the party in the two cities.

Q08. A trust caring for handicapped children gets ₹30000 every month from its donors. The trust spends half of the funds received for medical and educational care of the children and for that it charges 2% of the spent amount from them, and deposits the balance amount in a private bank to get the money multiplied so that in future the trust goes on functioning regularly. What percent of interest should the trust get from the bank to get a total of ₹1800 every month? Use matrix method, to find the rate of interest.

Q09. A trust fund has ₹35,000 is to be invested in two different types of bonds. The first bond pays 8% interest per annum which will be given to orphanage and second bond pays 10% interest per annum which will be given to an N.G.O. (cancer Aid Society).
Using matrix multiplication, determine how to divide ₹35,000 among two types of bonds if the trust fund obtains an annual total interest of ₹3,200.

Q10. A trust invested some money in two type of bonds. First bond pays 10% interest and second bond pays 12% interest. The trust received ₹2800 as interest.
However, if trust had interchanged money in bonds, they would have got ₹100 less as interest. Using matrix method, find the amount invested by the trust.

Q11. A typist charges ₹145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are ₹180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only ₹2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy?

Q12. A total amount of ₹7000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}$% respectively.
The total annual interest from these three accounts is ₹550. Equal amounts have been deposited in 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices.

Q13. Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand made fans, mats and plates from recycled material at a cost of ₹25, ₹100 and ₹50 each. The number of articles sold are given below :

Article \ School	A	B	C
Hand fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also find the total funds collected for the purpose.

Q14. There are 2 families A and B. There are 4 men, 6 women and 2 children in family A, and 2 men, 2 women and 4 children in family B. The recommended daily amount of calories is 2400 for men, 1900 for women, 1800 for children and 45 grams of protein for men, 55 grams for women and 33 grams for children.
Represent the above information using matrices. Using matrix multiplication, calculate the total requirement of calories and proteins for each of the 2 families.

Q15. To promote the making of toilets for women, an organization tried to generate awareness through (i) house calls (ii) letters, and (iii) announcements. The cost for each mode per attempt is given as :
(i) ₹50 (ii) ₹20 (iii) ₹40
The number of attempts made in three villages X, Y and Z are given below :

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organization for the three villages separately, using matrices.

Q16. Three schools A, B and C want to award their selected students for the values of Honesty, Regularity and Hard work. Each school decided to award a sum of ₹2500, ₹3100 and ₹5100 per student for the respective values. The number of students to be awarded by the three schools is given below in the table :

Values \ School	A	B	C
Honesty	3	4	6
Regularity	4	5	2
Hard work	6	3	4

Find the total money given in awards by the three schools separately, using matrices.

Q17. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹15,000 per month, find their monthly incomes using matrix method.

Q18. A coaching institute of English (subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is ₹9000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is ₹26,000.
Using matrix method, find monthly fees paid by each child of two types.

Q19. If a circle $(x-h)^2+(y-k)^2=r^2$ passes through the points (5, 7), (6, 6) and (2,–2), then find its centre (h, k), using matrix method. Also find the radius r.

Q20. The equation of the path traversed by the ball headed by the footballer is $y = ax^2 + bx + c$; (where $0 \le x \le 14$ and $a, b, c \in \mathbb{R}$ and $a \ne 0$) with respect to a XY-coordinate system in the vertical plane. The ball passes through the points $(2, 15)$, $(4, 25)$ and $(14, 15)$. Determine the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method. Also, find the equation of the path traversed by the ball.

Q21. The curve $y = ax^2 + bx + c$; (where $a, b, c \in \mathbb{R}$ and $a \ne 0$) passes through the points $(-1, 0)$, $(2, 12)$ and $(3, 20)$. Use matrix method to determine the values of a, b and c by solving the system of linear equations in a, b and c.

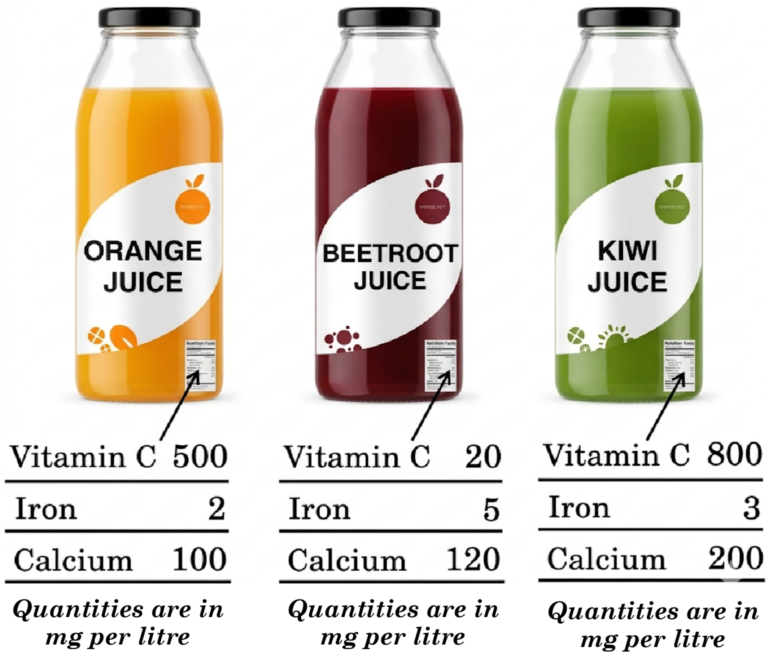
Find the equation of the curve. If $y = ax^2 + bx + c = 0$, then write the real roots of quadratic equation (if possible).

Q22. An insurance company agent Mr Raghav has the following record of policies sold in the month of April, May and June, 2024 for three different policies - Policy A, Policy B and Policy C.
Mr Raghav is paid a fixed commission per policy sold but the commission varies for the policies A, B and C.

Months	**Number of policies sold**			**Total commission earned (in ₹)**
	Policy A	Policy B	Policy C	
April	8	4	6	7850
May	9	9	6	9600
June	12	9	12	15000

Use matrix method to find the fixed commission payable on the policies A, B and C per unit.

Q23. Sravan is a nutritionist. He wants to create a mixture of orange juice, beetroot juice and kiwi juice that can provide 1860 mg of vitamin C, 22 mg of iron and 760 mg of calcium. The quantity of each nutrient per litre of juice is shown below.



Using matrix method, find how many litres of each juice Sravan should add into the mixture.

MATHEMATICIA BY O.P. GUPTA

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COMPETENCY
FOCUSED QUESTIONS

- **MULTIPLE CHOICE QUESTIONS**
- **ASSERTION-REASON QUESTIONS**
- **CASE-STUDY QUESTIONS**
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MULTIPLE CHOICE TYPE QUESTIONS

For CBSE 2027 Exams - Mathematics (041) - Class 12

Chapter 01 – *Matrices & Determinants*

Select the correct option (s) in the followings.

Q01. If A and B are two matrices such that $A+B$ and AB are both defined, then
(a) A and B can be any matrices
(b) A and B are square matrices not necessarily of same order
(c) Number of columns in A = Number of rows in B
(d) A and B are square matrices of same order.

Q02. If $A=\begin{bmatrix}2 & -1\\ 3 & 1\end{bmatrix}$ then, AA^T is equal to

(a) $\begin{bmatrix}5 & 5\\ 10 & 5\end{bmatrix}$ (b) $5\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$ (c) $5\ I_2$ (d) $\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$

Q03. Let $|A|=\begin{vmatrix}a & b & c\\ d & e & f\\ g & h & i\end{vmatrix}=4$. Then $|adj.A|=$

(a) 16 (b) 2 only (c) –2 only (d) –16

Q105. If $\begin{bmatrix}x & 2\\ 3 & x-1\end{bmatrix}$ is a singular matrix, then the product of all possible values of x is

(a) 6 (b) –6 (c) 0 (d) –7

Q106. If $\left|\dfrac{A^{-1}}{2}\right|=\dfrac{1}{k|A|}$, where A is a 3×3 matrix, then the value of k is

(a) $\dfrac{1}{8}$ (b) 8 (c) 2 (d) $\dfrac{1}{2}$

Q107. If $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, then A^{2023} is equal to

(a) $\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ (b) $\begin{bmatrix}0 & 2023\\ 0 & 0\end{bmatrix}$ (c) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ (d) $\begin{bmatrix}2023 & 0\\ 0 & 2023\end{bmatrix}$

Q108. If $\begin{bmatrix}2 & 0\\ 5 & 4\end{bmatrix}=P+Q$, where P is a symmetric and Q is a skew symmetric matrix, then $Q=$

(a) $\begin{bmatrix}2 & 5/2\\ 5/2 & 4\end{bmatrix}$ (b) $\begin{bmatrix}0 & -5/2\\ 5/2 & 0\end{bmatrix}$ (c) $\begin{bmatrix}0 & 5/2\\ -5/2 & 0\end{bmatrix}$ (d) $\begin{bmatrix}2 & -5/2\\ 5/2 & 4\end{bmatrix}$

Q110. If $|A|=|kA|$, where A is a square matrix of order 2, then sum of all possible value of k is
(a) 1 (b) –1 (c) 2 (d) 0

Q111. Number of symmetric matrices of order 3×3 with each entry 1 or –1 is
(a) 512 (b) 64 (c) 8 (d) 4

Q112. If $A=\begin{bmatrix}1 & 4 & x\\ z & 2 & y\\ -3 & -1 & 3\end{bmatrix}$ is a symmetric matrix, then the value of $x+y+z$ is

(a) 10 (b) 6 (c) 8 (d) 0

Q113. Let A be the area of a triangle having vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, x_3). Which of the following is correct?

(a) $\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\pm A$ (b) $\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\pm 2A$

(c) $\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\pm\dfrac{A}{2}$ (d) $\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}^2=A^2$

Q114. Let A be a skew-symmetric matrix of order 3. If $|A|=x$, then $(2023)^x$ is equal to

(a) 2023 (b) $\dfrac{1}{2023}$ (c) $(2023)^2$ (d) 1

Q116. The value of $\begin{vmatrix}x+y & y+z & z+x\\ z & x & y\\ 1 & 1 & 1\end{vmatrix}$ is

(a) 0 (b) 1 (c) $x+y+z$ (d) $2(x+y+z)$

Q120. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix}a & c & e\\ b & d & f\\ 1 & 1 & 1\end{vmatrix}^2$ is equal to

(a) $2\Delta^2$ (b) $4\Delta^2$ (c) 2Δ (d) 4Δ

Q157. If M is a diagonal matrix of order 3 with all the principal diagonal elements equal to p, where $p\neq 0$, then the determinant of $M^{-1}=$

(a) p^3 (b) 0 (c) 1 (d) p^{-3}

Q158. The number of all non-zero Scalar matrices of order 3, with each entry -1, 0 or 1, is

(a) 1 (b) 3 (c) 2 (d) 3^9

Chapter 02 - Relations & Functions

Select the correct option (s) in the followings.

Q01. The relation $R=\{(1,2)\}$ on $A=\{1,2,3\}$ is

(a) Reflexive only
(b) Symmetric only
(c) Transitive only
(d) Equivalence i.e., reflexive, symmetric and transitive

Q03. Let $f:A\to B$ be a one-one function s.t. range of f is $\{b\}$. Then the value of $n(A)$ is

(a) 1 (b) 2 (c) 0 (d) 4

Q14. For real numbers x and y, define xRy if and only if $x-y+\sqrt{2}$ is an irrational number. Then the relation R is
(a) only reflexive (b) only symmetric (c) only transitive (d) equivalence

Q36. Let R be the relation in the set N given by $R=\{(a,b): a=b-2,\ b>6\}$.
Which of the following is true?
(a) $(2,4)\in R$ (b) $(3,8)\in R$ (c) $(6,8)\in R$ (d) $(8,7)\in R$

Q37. If $f(x)=|\cos x|$, then $f\left(\frac{3\pi}{4}\right)$ is
(a) 1 (b) -1 (c) $-\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

Q46. A function $f:\mathbb{R}\to\mathbb{R}$ defined as $f(x)=x^2-4x+5$ is
(a) injective but not surjective (b) surjective but not injective
(c) both injective and surjective (d) neither injective nor surjective

Chapter 03 - Inverse Trigonometric Functions

📖 **Select the correct option (s) in the followings.**

Q01. The value of $\cos^{-1}(-1)-\sin^{-1}(1)$ is
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) $-\frac{3\pi}{2}$

Q20. If $\tan^{-1}x=\frac{\pi}{10}$, for some $x\in R$, then the value of $\cot^{-1}x$ is
(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) $\frac{4\pi}{5}$

Q23. The domain of the function defined by $f(x)=\sin^{-1}x+\cos x$ is
(a) $[-1,\ 1]$ (b) $[-1,\ \pi+1]$ (c) $(-\infty,\infty)$ (d) ϕ

Q28. The range of $f(x)=\frac{1}{2}\sin^{-1}2x$ is
(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (b) $[-1,\ 1]$ (c) $\left\{\frac{\pi}{2}\right\}$ (d) $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$

Q35. If $y=\sin x$ is invertible i.e., inverse of $y=\sin x$ exists, then which of the following is correct?
(a) $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right),\ y\in[-1,\ 1]$ (b) $x\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right],\ y\in[-1,\ 1]$
(c) $x\in[-1,\ 1],\ y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (d) $x\in R,\ y\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$

Q37. Let $m=\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ and $n=\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$. Then $(m+n)^{\frac{\pi}{4}+n}=$
(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

Q38. If $\frac{\pi}{4}<x<\frac{\pi}{2}$, then $\tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)=$

(a) $\frac{\pi}{4}+x$ (b) $\frac{\pi}{4}-x$ (c) $\frac{3\pi}{4}-x$ (d) $x-\frac{3\pi}{4}$

Q41. If $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\frac{\pi}{3}$, then the value of k is

(a) 1 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Chapter 04 - Continuity & Differentiability

Select the correct option (s) in the followings.

Q01. Value of $\frac{d}{dx}\left(\sin^{-1}\frac{x}{3}+\cos^{-1}\frac{x}{3}\right)$ is equal to

(a) 0 (b) $\frac{1}{3}$ (c) 3 (d) Not possible to find

Q17. The derivative of $|x|$ at $x\neq 0$

(a) is 1 (b) is -1 (c) is 0 (d) is ± 1

Q18. Consider the following statements :

I: $\lim_{x\to 0}\sin\frac{1}{x}$ doesn't exist.

II: $\lim_{x\to 0}x\sin\frac{1}{x}$ exists.

Which of the above statements is/are correct?

(a) I only (b) II only (c) Both I and II (d) Neither I nor II

Q52. If $f(x)=2|x|+3|\sin x|+6$, then the right hand derivative of $f(x)$ at $x=0$ is

(a) 6 (b) 5 (c) 3 (d) 2

Q54. The function $f(x)=x|x|$ is

(a) continuous and differentiable at $x=0$
(b) continuous but not differentiable at $x=0$
(c) differentiable but not continuous at $x=0$
(d) neither differentiable nor continuous at $x=0$

Q55. If $\tan\left(\frac{x+y}{x-y}\right)=k$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{y}{x}$ (b) $\frac{y}{x}$ (c) $\sec^2\left(\frac{y}{x}\right)$ (d) $-\sec^2\left(\frac{y}{x}\right)$

Q58. Let $f(x)=x-[x]$, where [.] is a g.i.f. Then find $f'\left(\frac{1}{2}\right)=$

(a) not defined (b) 0 (c) 1 (d) -1

Q78. If $y=\sqrt{\cos x+y}$ gives $\frac{dy}{dx}=\frac{\sin x}{k-2y}$, then $k=$

(a) 1 (b) -1 (c) 2 (d) -2

Q79. If $y=A\sin 2x+B\cos 2x$ and $\frac{d^2y}{dx^2}-ky=0$, then the value of k is

(a) 4 (b) $-\frac{1}{4}$ (c) –4 (d) $\frac{1}{4}$

Chapter 05 - Applications Of Derivatives

Select the correct option (s) in the followings.

Q01. If $f(x)=\log x$, then $f(x)$ is

(a) always increasing
(b) always decreasing
(c) both increasing and decreasing
(d) neither increasing nor decreasing

Q43. The maximum value of xy, if $x+2y=8$, is

(a) 8 (b) 16 (c) 20 (d) 24

Q45. The rate of change of the surface area of the sphere of radius r when the radius is increasing at the rate of 2 cm/s is proportional to

(a) $\frac{1}{r^2}$ (b) $\frac{1}{r}$ (c) r (d) r^2

Q47. The rate of change of the volume of sphere with respect to its surface area, when its radius is 2 units, is

(a) 1 (b) 2 (c) 3 (d) 4

Q48. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which the area increases, when side is 10 cm is

(a) 10 cm^2/s (b) $\sqrt{3}\ cm^2/s$ (c) $10\sqrt{3}\ cm^2/s$ (d) 10/3 cm^2/s

Q49. If $f(x)=a(x-\cos x)$ is strictly decreasing in $\mathbb{R}$, then 'a' belongs to

(a) {0} (b) $(0,\infty)$ (c) $(-\infty,0)$ (d) $(-\infty,\infty)$

Chapter 06 - Indefinite Integrals

Select the correct option (s) in the followings.

Q01. If $\int e^{-2\log x}dx=f(x)+k$, then $f(x)$ is

(a) $\frac{x^3}{3}$ (b) $-\frac{1}{x}$ (c) $-\frac{2}{x}$ (d) $\frac{1}{x}$

Q41. If $\frac{d}{dx}[f(x)]=ax+b$ and $f(0)=0$, then $f(x)$ is equal to

(a) $a+b$ (b) $\frac{ax^2}{2}+bx$ (c) $\frac{ax^2}{2}+bx+c$ (d) b

Q43. Anti-derivative of $\frac{\tan x-1}{\tan x+1}$ with respect to x is

(a) $\sec^2\left(\frac{\pi}{4}-x\right)+c$ (b) $-\sec^2\left(\frac{\pi}{4}-x\right)+c$

(c) $\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|+c$ (d) $-\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|+c$

Q44. $\int \frac{2\cos 2x - 1}{1 + 2\sin x} dx$ is equal to

(a) $x - 2\cos x + C$ (b) $x + 2\cos x + C$ (c) $-x - 2\cos x + C$ (d) $-x + 2\cos x + C$

Q45. $\int \frac{\sec x}{\sec x - \tan x} dx$ equals

(a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$ (c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$

Chapter 07 - *Definite Integrals*

Select the correct option (s) in the followings.

Q01. If $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2} = ay$, then the value of a is

(a) 9 (b) 5 (c) –9 (d) –5

Q37. Value of $\int_0^2 \frac{dx}{x^2 + 4}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) None of these

Q39. If $\int_0^{2\pi} \cos^2 x\, dx = k \int_0^{\pi/2} \cos^2 x\, dx$, then the value of k is

(a) 4 (b) 2 (c) 1 (d) 0

Q40. If $\int_0^a 3x^2 dx = 8$, then the value of 'a' is

(a) 2 (b) 4 (c) 8 (d) 10

Q41. $\int_{-1}^{1} \frac{|x-2|}{x-2} dx,\ x \neq 2$ is equal to

(a) 1 (b) –1 (c) 2 (d) –2

Chapter 08 - *Application Of Integrals*

Select the correct option (s) in the followings.

Q01. The area of the ellipse whose major axis is on the x-axis, is

(a) $\pi a b$ (b) $\pi(a+b)$ (c) $\frac{\pi}{4}(a^2+b^2)$ (d) $\frac{\pi}{4}(ab)$

Q02. Area of the triangle (in square units) bounded by the sides $x = 0$, $y = 0$ and $x + y = 2$ is

(a) 1 (b) 2 (c) 4 (d) 8

Q26. The area cut off from the parabola $y^2 = px$ by the line $y = px$ is

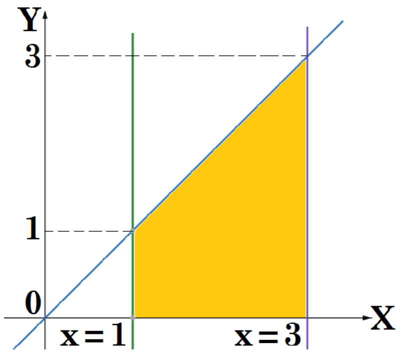
(a) $\frac{p}{6}$ (b) $\frac{1}{6p}$ (c) $\frac{p^2}{2}$ (d) $\frac{p^3}{3}$

Q27. The area of the region bounded by the curves $y = x^2$ and $y = |x|$ is

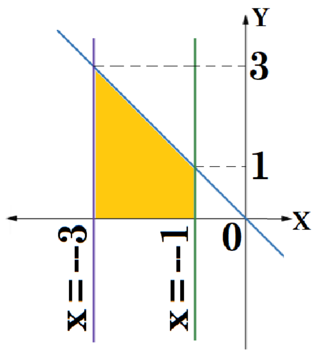
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{5}{3}$

Q33. Which of the following graph gives the area represented by $\int_1^3 x\,dx$?

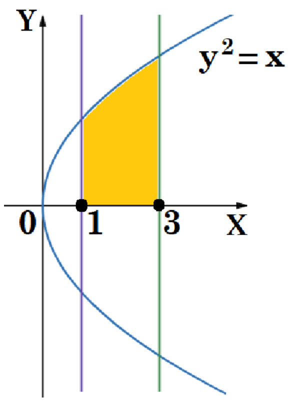
(a)



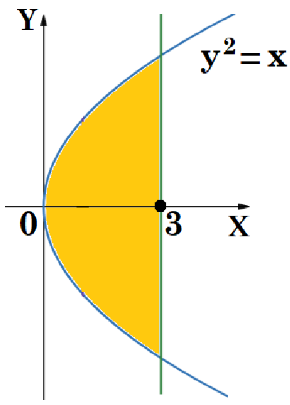
(b)



(c)



(d)



Chapter 09 – Differential Equations

📖 **Select the correct option (s) in the followings.**

Q01. The general solution of the differential equation $\ln\left(\frac{dy}{dx}\right) + x = 0$ is

(a) $y = e^{-x} + c$ (b) $y = -e^{-x} + c$ (c) $y = e^{x} + c$ (d) $y = -e^{x} + c$

Q31. The solution of the differential equation $\cos x \cos y dx + \sin x \sin y dy = 0$ is

(a) $\tan x = c$ (b) $\sec x - \sec y = c$ (c) $\sec y . \sin x = c$ (d) $\text{cosec}\, y . \cos x = c$

Q32. The slope a curve at any point, is the reciprocal of twice the ordinate and it passes through (4, 3). The equation of the curve is

(a) $y^2 - x + 5 = 0$ (b) $x^2 - y + 5 = 0$ (c) $y^2 - x - 5 = 0$ (d) $x^2 - y - 5 = 0$

Q34. The integrating factor of the differential equation $(1-y^2)\dfrac{dx}{dy}+yx=ay,\ (-1<y<1)$ is

(a) $\dfrac{1}{y^2-1}$ (b) $\dfrac{1}{\sqrt{y^2-1}}$ (c) $\dfrac{1}{1-y^2}$ (d) $\dfrac{1}{\sqrt{1-y^2}}$

Q35. The number of solutions of the differential equation $\dfrac{dy}{dx}=\dfrac{y+1}{x-1}$, when $y(1)=2$, is

(a) zero (b) one (c) two (d) infinite

Chapter 10 – Linear Programming

Select the correct option (s) in the followings.

Q01. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z=px+qy$, where $p,q>0$. Condition on p and q so that the maximum of Z occurs at the points (15, 15) and (0, 20) both, is

(a) $p=q$ (b) $p=2q$ (c) $q=2p$ (d) $q=3p$

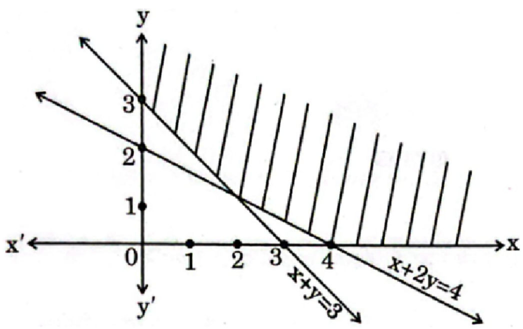
Q45. The number of feasible solutions of the linear programming problem given as

Maximize $z=15x+30y$

Subject to constraints $3x+y\le 12$, $x+2y\le 10$, $x\ge 0$, $y\ge 0$ is

(a) 1 (b) 2 (c) 3 (d) infinite

Q46. The feasible region of a linear programming problem is shown in the figure below



Which of the following are the possible constraints?

(a) $x+2y\ge 4,\ x+y\le 3,\ x\ge 0,\ y\ge 0$

(b) $x+2y\le 4,\ x+y\le 3,\ x\ge 0,\ y\ge 0$

(c) $x+2y\ge 4,\ x+y\ge 3,\ x\ge 0,\ y\ge 0$

(d) $x+2y\ge 4,\ x+y\ge 3,\ x\le 0,\ y\le 0$

Q48. Which of the following points satisfies both the inequalities $2x+y\le 10$ and $x+2y\ge 8$?

(a) (–2, 4) (b) (3, 2) (c) (–5, 6) (d) (4, 2)

Chapter 11 – Vector Algebra

Select the correct option (s) in the followings.

Q01. The magnitude of the vector $6\hat{i}+2\hat{j}+3\hat{k}$ is

(a) 5 (b) 7 (c) 12 (d) 1

Q64. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

Q65. In ΔABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD} =$

(a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$ (c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$

Q66. All the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$, are given by

(a) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (b) $-\left(\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right)$ (c) $\pm\left(\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right)$ (d) $\pm\left(\frac{\hat{i} + \hat{j} + \hat{k}}{3}\right)$

Q67. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vectors. Then angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is

(a) 90° (b) 180° (c) 45° (d) 0°

Chapter 12 - *Three Dimensional Geometry*

Select the correct option (s) in the followings.

Q01. Distance of the point (α, β, γ) from y-axis is

(a) β (b) $|\beta|$ (c) $|\beta| + |\gamma|$ (d) $\sqrt{\alpha^2 + \gamma^2}$

Q30. The value of λ for which the angle between the lines

$\vec{r} = \hat{i} + \hat{j} + \hat{k} + p(2\hat{i} + \hat{j} + 2\hat{k})$ and $\vec{r} = (1+q)\hat{i} + (1+q\lambda)\hat{j} + (1+q)\hat{k}$ is $\frac{\pi}{2}$, is

(a) -4 (b) 4 (c) 2 (d) -2

Q32. If the direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$, then

(a) $0 < a < 1$ (b) $a > 2$ (c) $a > 0$ (d) $a = \pm\sqrt{3}$

Q33. The point $(x, y, 0)$ on the xy-plane divides the line segment joining the points $(1, 2, 3)$ and $(3, 2, 1)$ in the ratio

(a) $1:2$ internally (b) $2:1$ internally (c) $3:1$ internally (d) $3:1$ externally

Q34. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) 0° (b) 30° (c) 45° (d) 90°

Chapter 13 - *Probability*

Select the correct option (s) in the followings.

Q01. If A and B are independent events and $P(A \cup B) = \frac{3}{8}$, then $P(A').P(B')$ is

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{8}$ (d) $\frac{7}{8}$

Q35. One mapping (i.e., function) is selected at random from all the mappings of the set $A = \{1, 2, 3, 4, 5, 6\}$ into itself. Then, the probability that the mapping selected is a one-one mapping, is

(a) $\frac{5}{324}$ (b) $\frac{4}{325}$ (c) $\frac{2}{354}$ (d) $\frac{3}{524}$

Q45. If A and B are two events such that $P(A \mid B) = 2 \times P(B \mid A)$ and $P(A) + P(B) = \frac{2}{3}$, then $P(B)$ is equal to

(a) $\frac{2}{9}$ (b) $\frac{7}{9}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

Q53. Five fair coins are tossed simultaneously. The probability of the events that at least one head comes up is

(a) $\frac{27}{32}$ (b) $\frac{5}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{32}$

Q63. A matrix B of order 2 is randomly selected from all the matrices of order 2×2 with entries 0 or 1. What is the probability of matrix B to be a diagonal matrix?

(a) $\frac{1}{8}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

ASSERTION REASON TYPE QUESTIONS

For CBSE 2027 Exams - Mathematics (041) - Class 12

By O.P. GUPTA
Indira Award Winner
M.+919650350480

In the following questions, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Unit 1 (Relations & Functions)

Relations & Functions, Inverse Trig. Functions

Q01. **Assertion (A) :** The relation $R = \{(a, b) : a \le b^2\}$ on the set R of real nos. is not reflexive.

Reason (R) : A relation on a set A is reflexive if $(a, a) \in R \; \forall \; a \in A$.

Q07. **Assertion (A) :** Let $f : R \to R$ defined as $f(x) = [\,x\,]$, here $[\,.\,]$ represents the greatest integer function. Then f is not one-one.

Reason (R) : A function is one-one if $f(\alpha) = f(\beta)$ implies $\alpha = \beta$.

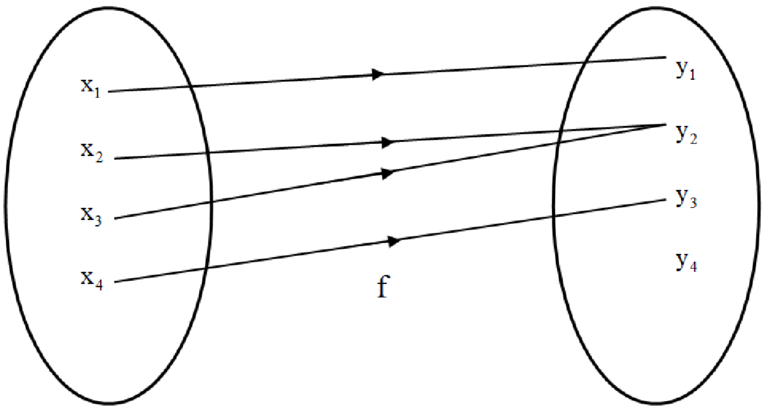
Q11. **Assertion (A) :** Number of all onto functions from the set {1, 2, 3, 4} to itself is 24.

Reason (R) : Onto functions from the set {1, 2, 3, …, n} to itself is simply a **permutation** on n symbols namely 1, 2, 3, …, n.

Q27. **Assertion (A) :** Inverse of $\sin x$ does not exist in $x \in R$.

Reason (R) : All trigonometric functions are many-one in their respective domain.

Q30. **Assertion (A) :** A function f shown below by the arrow diagram, is one-one.



Reason (R) : A function $f : A \to B$ is one-one if $f(\alpha) = f(\beta)$ implies $\alpha = \beta$ for all $\alpha, \beta \in A$.

Q38. $X = \{0, 2, 4, 6, 8\}$.

P is a relation on X defined by $P = \{(0, 2), (4, 2), (4, 6), (8, 6), (2, 4), (0, 4)\}$.

Assertion (A) : The relation P on set X is a transitive relation.

Reason (R) : The relation P has a subset of the form $\{(a, b), (b, c), (a, c)\}$, where $a, b, c \in X$.

Q40. **Assertion (A) :** Domain of $y=\cos^{-1}(x)$ is $[-1, 1]$.

Reason (R) : The range of the principal value branch of $y=\cos^{-1}(x)$ is $[0, \pi]-\left\{\frac{\pi}{2}\right\}$.

Unit 2 (Algebra)
Matrices, Determinants

Q01. Let A and B be two symmetric matrices of order 3.

Assertion (A) : A(BA) and (AB)A are symmetric matrices.

Reason (R) : AB is symmetric matrix if matrix multiplication of A with B is commutative.

Q06. **Assertion (A) :** The inverse of $A=\begin{pmatrix}5 & 1\\ 2 & 2\end{pmatrix}$ does not exist.

Reason (R) : Matrix A is non-singular.

Q30. **Assertion (A) :** If $A=\begin{pmatrix}-2 & 3\\ 4 & 9\end{pmatrix}$ and $B=\begin{pmatrix}1 & 4 & 5\\ 6 & 7 & 8\end{pmatrix}$ then the product AB is of order 2×3.

Reason (R) : For a null matrix, all of its elements are zero.

Q33. **Assertion (A) :** Matrix $M=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$ is a diagonal matrix.

Reason (R) : A diagonal matrix is a square matrix, in which all the non-diagonal elements are zero.

Q34. **Assertion (A) :** For matrix $M=\begin{bmatrix}4 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 2\end{bmatrix}$, determinant value $|M|=24$.

Reason (R) : For a diagonal matrix $\text{diag.}\begin{pmatrix}a & b & c\end{pmatrix}$, the det. value is given by 'abc'.

Q40. **Assertion (A) :** A system of three linear equations in three variables always has a unique solution if the determinant of the coefficient matrix is non-zero.

Reason (R) : For a diagonal matrix $X=\begin{pmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{pmatrix}$, $\det.(X)=0$.

Unit 3 (Calculus)
Continuity & Differentiability, Applications of Derivatives, Integrals, Application of Integrals, Differential Equations

Q01. **Assertion (A) :** $f(x)=\log|x|$ is always continuous for all real values of x.

Reason (R) : A function is always continuous at all the points of its domain.

Q19. **Assertion (A) :** If $y = x e^x$, then $\frac{dy}{dx} = x e^x + e^x$.

Reason (R) : $\frac{d}{dx}(u v) = u\frac{dv}{dx} + v\frac{du}{dx}$.

Q25. **Assertion (A) :** $f(x) = \sin 2x + 3$ is defined for all real values of x.

Reason (R) : Maximum value of $f(x)$ is 4 and minimum value is 2.

Q42. **Assertion (A) :** Order of differential equation $\log\left(\frac{d^2y}{dx^2}\right) = \left(\frac{dy}{dx}\right)^3 + x$ is 2.

Reason (R) : Order of a differential equation is the order of the highest order derivative appearing in the differential equation.

Q52. **Assertion (A) :** For $y = 9x^2$, $x = 0$, $y = 1$ and $y = 4$, the area of the closed region in the first quadrant is $\frac{14}{9}$ Sq. units.

Reason (R) : For the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, the area enclosed in first quadrant is 6π Sq. units.

Q60. **Assertion (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx = 3$.

Reason (R) : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$.

Q62. **Assertion (A) :** The maximum value of the function $f(x) = x^5$, where $x \in [-1, 1]$ is attained at its critical point, $x = 0$.

Reason (R) : The maximum of a function can only occur at points where derivative is zero.

Unit 4 (Vectors & 3 D Geometry)
Vector Algebra, Three Dimensional Geometry

Q01. **Assertion (A) :** Value of $\lambda = 2$, if $\vec{a} \| \vec{b}$, where $\vec{a} = 2\hat{i} + 4\hat{j} + 3\lambda\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

Reason (R) : $\vec{a}.\vec{b} = 0$ implies, $\vec{a} \perp \vec{b}$, if $\vec{a}$ and $\vec{b}$ are non-zero vectors.

Q16. **Assertion (A) :** If $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ then, $\vec{a}.\vec{a} = 29$.

Reason (R) : $\vec{a}.\vec{a} = |\vec{a}|^2$.

Q25. **Assertion (A) :** $\vec{r} = \hat{i} + \lambda(2\hat{i} + 3\hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(3\hat{i} + \hat{j} - 2\hat{k})$ are perpendicular lines.

Reason (R) : For two perpendicular lines whose d.r.'s are a_1, b_1, c_1 and a_2, b_2, c_2, we must have $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

Q30. **Assertion (A) :** The shortest distance between the lines $\vec{r} = 8\hat{i} - 9\hat{j} + 10\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$ is given by 14 units.

Reason (R) : The shortest distance between the parallel lines $\vec{r}=\vec{a}_1+\lambda\vec{b}$ and $\vec{r}=\vec{a}_2+\mu\vec{b}$, is given by $S.D.=\frac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}$.

Q32. **Assertion (A) :** The lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ are perpendicular, when $\vec{b}_1\cdot\vec{b}_2=0$.

Reason (R) : The angle θ between the lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ is given by the expression $\cos\theta=\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$.

Q36. **Assertion (A) :** The acute angle between the line $\vec{r}=\hat{i}+\hat{j}+2\hat{k}+\lambda(\hat{i}-\hat{j})$ and the x-axis is 45^o.

Reason (R) : The acute angle θ between the lines $\vec{r}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}+\lambda(a_1\hat{i}+b_1\hat{j}+c_1\hat{k})$ and $\vec{r}=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}+\mu(a_2\hat{i}+b_2\hat{j}+c_2\hat{k})$ is given by $\cos\theta=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$.

Q45. **Assertion (A) :** For a vector $\vec{p}$, we always have $\vec{p}.\vec{p}=\vec{0}$.

Reason (R) : If $\overrightarrow{OA}=\vec{x}$ and $\overrightarrow{OB}=\vec{y}$, then $\overrightarrow{BA}=\vec{x}-\vec{y}$.

Unit 5 (Linear Programming)
Linear Programming Problems

Q01. **Assertion (A) :** In a particular LPP whose objective function is given as $Z=x+y$, the corner points of the feasible region are found to be (25, 0), (0, 40) and (0, 0) and so, $Z_{max}=40$.

Reason (R) : The maximum or minimum values of objective function occur at the corner point of the feasible region.

Q08. Corner points of a LPP are given as O(0, 0), A(7, 0), B(3, 4) and C(0, 2).

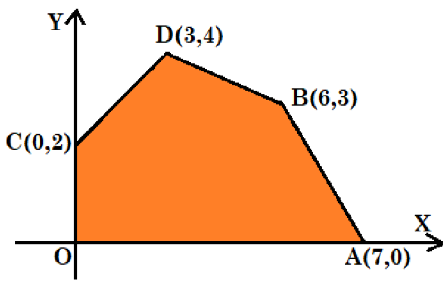
Assertion (A) : Let $Z=px+y$ and $Z_A=Z_C$ then, the value of $p=\frac{2}{7}$.

Reason (R) : If $Z=qx+y$ and $Z_A=2Z_C$, then value of $q=\frac{3}{7}$.

Q09. The corner points of the feasible region determined by the system of linear constraints are as shown below.

Assertion (A) : Let $Z=x+2y$ be the objective function. Then maximum value of Z occurs at B(6, 3).

Reason (R) : For the objective function $Z=x+2y$, $Z_{max}=11$.



Unit 6 (Probability)
Probability

Q01. **Assertion (A) :** If $P(A)=\frac{2}{5}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{5}$, then $P(\bar{A}\,|\,\bar{B})=\frac{7}{10}$.

Reason (R) : $P(\bar{A}\,|\,\bar{B})=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$, $P(\overline{A\cup B})=P(\bar{A}\cap\bar{B})$ and $P(\bar{E})=1-P(E)$.

Q17. **Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment, then $P(F\,|\,E)=\frac{P(E\cap F)}{P(E)}$.

Q20. If each element of a second order determinant is either 0 or 1, then the probability that the value of the determinant is positive is given by P. Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$.

Assertion (A) : $P=\frac{3}{16}$.

Reason (R) : For $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the value of det. (A) is given by expression $ad+bc$.

Q23. **Assertion (A) :** Suppose a function $f:\mathbb{R}\to\mathbb{R}$ is defined by

$$f(x)=\begin{cases} x^2, \text{ if } x\le 1 \\ 2x-1, \text{ if } x>1 \end{cases}.$$

Then, the probability that f is continuous at $x=1$ is zero.

Reason (R) : For a function to be continuous at a point, its left-hand limit, right-hand limit, and value at that point must be equal.

Application of Integrals

Differential Equations

Vector Algebra

3 D Geometry

Linear Programming

Probability

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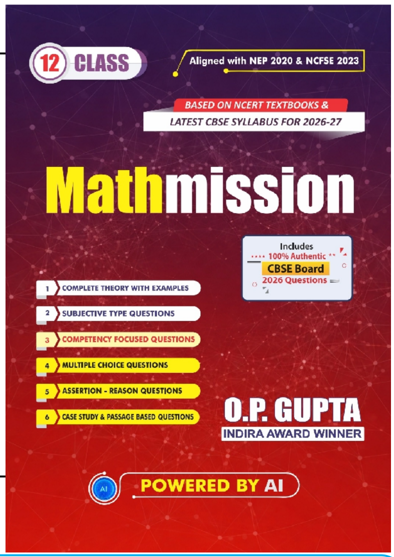

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CASE STUDY &
PASSAGE BASED Questions

Useful for CBSE Exams 2026-27

🕮 Unit I - Relations & Functions

☑ Relations & Functions, Inverse Trig. Functions

Q01. In two different societies, there are some school going students - including girls as well as boys. Satish forms two sets with these students, as his college project.

Let $A=\{a_1, a_2, a_3, a_4, a_5\}$ and $B=\{b_1, b_2, b_3, b_4\}$ where a_i's and b_i's are the school going students of first and second society respectively.
Satish decides to explore these sets for various types of relations and functions.

Using the information given above, answer the following.

(i) Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible?

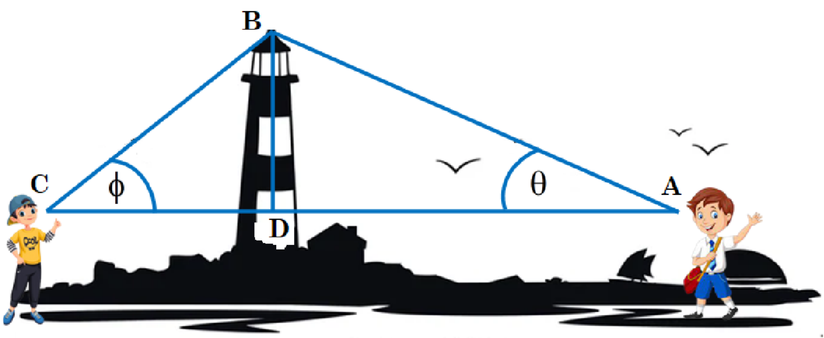
(ii) Let $R: A \to A$, where $R=\{(x, y): \text{x and y are students of same sex}\}$. Is the relation R an equivalence relation? Justify.

(iii) Satish and his friend Rajat are interested to know the number of symmetric relations defined on both the sets A and B, separately. Satish decides to find the symmetric relation on set A, while Rajat decides to find the symmetric relation on set B. What is difference between their results?

(iv) Let $R: A \to B$, $R=\{(a_1,b_1), (a_1,b_2), (a_2,b_1), (a_3,b_3), (a_4,b_2), (a_5,b_2)\}$. Then, is R onto or one-one or both or none? Justify.

(v) To help Satish in his project, Rajat decides to form onto function from set A to B. How many such functions are possible?

Q06. Rahul and Priyam are students of class XII.
They are standing on either side of a light-house of 20 meters high.
Rahul is standing at A and Priyam is at C. They observe its top at the angles of elevation θ and ϕ respectively (as shown in the figure below).



The distance between the two students is 30 meters and the distance between Rahul and the light-house is 20 meters.
Based on the above information, answer the following.

(i) Find $\theta = \angle CAB$.

(ii) Find the distance AB (as shown in the figure).

(iii) Find $\phi = \angle BCA$.

(iv) Find $\angle ABC$.

Q07. A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever.

ONE - NATION
ONE - ELECTION

FESTIVAL OF DEMOCRACY

GENERAL ELECTION - 2019

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows.

$R = \{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election - 2019$\}$.

Based on the above information, answer the following.

(i) Two friends X and $Y \in I$.
X and Y both exercised their voting right in the general election - 2019.
Then, state if $(X, Y) \in R$ is true or not. Give reason.

(ii) Mr. 'H' and his wife 'W' both exercised their voting right in general election - 2019.
Then, state if the following statement is true or not. Give reason.
"If $(H, W) \in R$ then, we may or may not have $(W, H) \in R$."

(iii) Check if R is reflexive or, symmetric. Give reasons to support your answer.

(iv) Mr. Ghanshyam exercised his voting right in general election - 2019.
While his brother (having voting right), Mr. Radheshyam went to have fun at a nearby mall. Can we have (Ghanshyam, Radheshyam) $\in$ R ? Give reason.
If Miss. Radhika (having voting right) goes with Mr. Radheshyam to the mall skipping the voting exercise, then is it correct to say (Radhika, Radheshyam) $\notin$ R ? Give reason.

Q15. Pratibha Vikas is an innovative program by the Government of Delhi, where cultural and literacy competitions are held between schools at cluster, block, district and state levels.

One of those competitions - Yogasana, is conducted under two categories : Middle school and High school.
From South Delhi district, three students from middle school and two students from high school were selected for the state level.

Let $M = \{m_1, m_2, m_3\}$ and $H = \{h_1, h_2\}$, represent the set of students from middle school and high school respectively who got selected for the state level from that district.

A relation $R: M \to M$ is defined by $R = \{(x, y): x \text{ and } y \text{ are students from the same category}\}$.

On the basis of the above information, answer the following questions.

(i) Check if the relation R is reflexive. Justify your answer.

(ii) Check if the relation R is symmetric. Justify your answer.

(iii) Check if the relation R is transitive. Is R an equivalence relation? Justify your answer.

(iv) Let a function $f: M \to H$ is defined as $f = \{(m_1, h_1), (m_2, h_2), (m_3, h_2)\}$.
Check whether the function f is one-one and onto. Justify your answer.

🕮 Unit II - Algebra

☑ Matrices, Determinants

Q01. Two farmers Ramkrishna and Hari Prasad cultivated three varieties of rice namely Basmati, Permal and Naura.


Basmati


Permal


Naura

The sale (in Rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices 'A' and 'B' :

September Sales (in Rupees) — Basmati Permal Naura; October Sales (in Rupees) — Basmati Permal Naura

$$A = \begin{pmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \text{Ramkrishna} \\ \text{Hari Prasad} \end{matrix} \quad \text{and, } B = \begin{pmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{pmatrix} \begin{matrix} \text{Ramkrishna} \\ \text{Hari Prasad} \end{matrix}$$

Based on the above information, answer the following.

(i) Write the matrix, which represents the combined sale in September and October for each farmer in each variety.

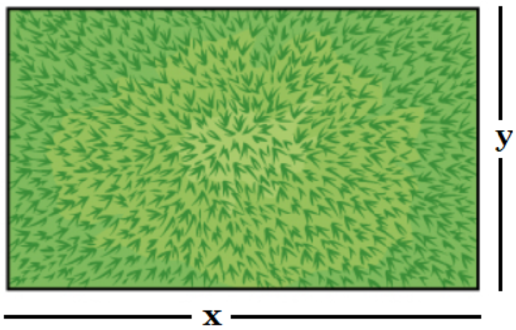
(ii) Write the matrix, which represents the decrease in sales from September to October.

(iii) If the farmer Hari Prasad receives 2% profit on gross sales, then find the total profit obtained in October.

(iv) If Ramkrishna receives 2% profit on gross sales, then find the total profit obtained in the month of October.

(v) What is the difference in the total profit earned by both the farmers in the month of September, if both the farmers receive 2% profit on gross sales?

Q08. Manjit wants to donate a rectangular plot of land for a school in his village.



When he was asked to give dimensions of the plot, he told that :

- If its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same,
- If length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.

For the information given above, answer the following.

(i) Assume that the length and breadth of the land be x and y (in metres) respectively. Find the equations in terms of x and y.

(ii) Using matrices, represent the linear equations obtained above in **(i)**.

(iii) Using matrices, determine the dimensions of the land (in metres). Also write the area of the rectangular plot of land (in square metres).

(iv) Suppose that, Manjit gave the information about his plot in the following manner :
If its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain the same, but if length is decreased by 20 m and breadth is decreased by 10 m, then its area will be decreased by 4800 m^2. In this situation, what will be dimensions of the plot? Assume that the length and breadth of the land be x and y (in metres) respectively. Use matrices.

Q09. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹250.
Based on the above information, answer the following questions.

(i) Convert the given above situation into a matrix equation of the form $AX = B$.

(ii) Find $|A|$.

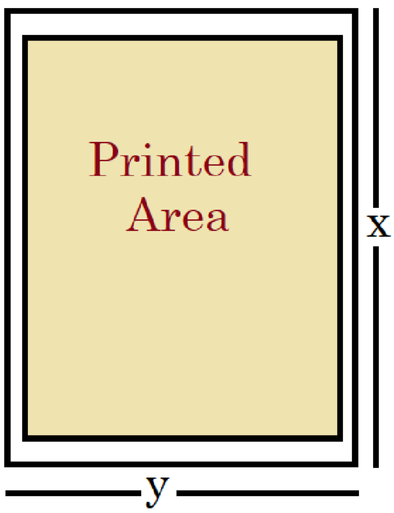
(iii) Find A^{-1}.

(iv) Determine $P = A^2 - 5A$.

🕮 Unit III - Calculus

☑ Continuity & Differentiability, Applications Of Derivatives, Integrals, Application Of Integrals, Differential Equations

Q01. Following is the pictorial description for a particular page, selected by a school administration.



The total area of the page is 150 cm^2.
The combined width of the margin at the top and bottom is 3 cm and the side 2 cm.
Using the information given above, answer the following.

(i) Find the relation between x and y.

(ii) Find the area of page where printing can be done.

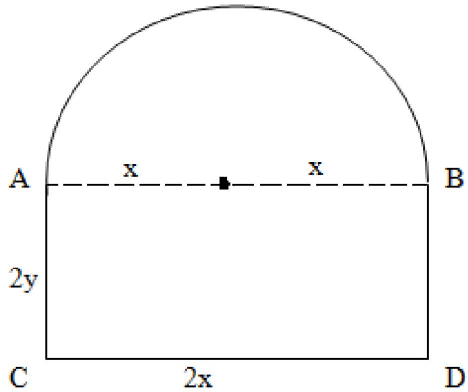
(iii) Find the area of the printable region of the page, in terms of x.

(iv) For what value of 'x', the printable area of the page is maximum? Use derivatives.

(v) What should be dimension of the page so that it has maximum area to be printed?

Q02. Mr Shashi, who is an architect, designs a building for a small company.
The design of window on the ground floor is proposed to be different than other floors.
The window is in the shape of a rectangle which is surmounted by a semi-circular opening.

This window is having a perimeter of 10 m as shown below.



Based on the above information answer the following.

(i) If 2x and 2y represents the length and breadth of the rectangular portion of the windows, then find the relation between the variables x and y.

(ii) Find the combined area (A) of the rectangular region and semi-circular region of the window expressed as a function of x.

(iii) Find the maximum value of area A, of the whole window.

(iv) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible.
For this to happen, find the length of rectangular portion of the window.

(v) In order to get the maximum light input through the whole window, find the area (in terms of square meter) of only semi-circular opening of the window.

Q13. A fighter-jet of enemy is flying along the parabolic path $y = x^2 + 7$.

A soldier is assigned duty to shoot down the fighter-jet.

Based on above information, answer the following.

(i) Assume that the soldier has located himself safely at a point (3, 7). If he decides to shoot down the fighter-jet when it is nearest to him, then find the function f (x) which determines the distance between the soldier and fighter-jet.

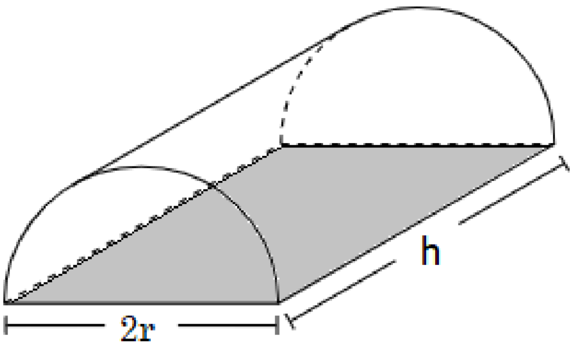
(ii) If $u = [f(x)]^2$ then, find $\frac{du}{dx}$.

(iii) Write $\frac{d^2u}{dx^2}$.

(iv) When the soldier shoots the fighter-jet, then find the distance between him and the fighter-jet at that instant.

(v) What will be the position of fighter-jet on the parabolic path, when the soldier shoots it down?

Q21. A company deals in casting and molding of metal on order received from its clients.
A given quantity of metal (1000 cubic units) is to be cast into a half cylinder with a rectangular base and semicircular ends.



Using the information given above, answer the following.

(i) Write an express for 'h', in terms of 'r'.

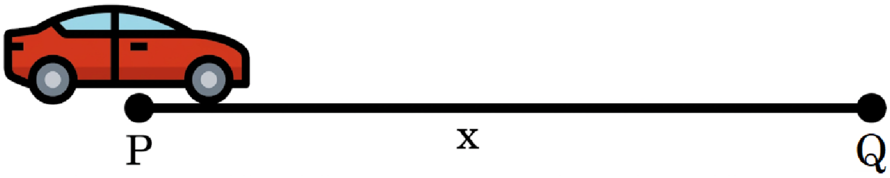
(ii) Express the total surface area (A) of the half-cylinder, in terms of 'r'.

(iii) Find $\frac{dA}{dr}$.

(iv) For what value of r, the total surface area (A) will be minimum?

(v) What is the value of $h:(2r)$?

Q29. A car starts from a point P at time $t=0$ seconds and stops at point Q.



The distance x, in the metres, covered by it, in t seconds is given by $x=t^2\left(2-\frac{t}{3}\right)$.

Based on the given information, answer the following.

(i) Find the time taken by the car to reach Q. Use derivatives.

(ii) Find the distance between the points P and Q.

Q36. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.

A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2\text{ cm}^3/\text{s}$.

The semi-vertical angle of the conical tank is $45°$.

On the basis of given information, answer the following questions.

(i) Find the volume of water in the tank in terms of its radius r.

(ii) Find rate of change of radius at an instant when $r=2\sqrt{2}$ cm.

(iii) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r=2\sqrt{2}$ cm.

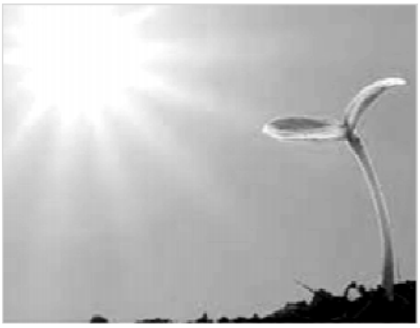
(iv) Find the rate of change of height 'h' at an instant when slant height is 4 cm.

Q41. The equation of the path traced by a roller-coaster is given by the polynomial $f(x)=a(x+9)(x+1)(x-3)$.

If the roller-coaster crosses y-axis at a point $(0,-1)$, answer the following questions.

(i) Find the value of 'a'.
(ii) Find $f''(x)$ at $x = 1$.

Q42. The relation between the height of the plant ('y' in cm) with respect to its exposure to the sunlight is governed by the following equation
$y = 4x - \frac{1}{2}x^2$, where 'x' is the number of days exposed to the sunlight, for $x \le 3$.



Based on the above information, answer the following.

(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

(ii) Does the rate of growth of the plant increase or decrease in the first three days? What will be the height of the plant after 2 days?

Q54. Ramesh, the owner of a sweet selling shop, purchased some rectangular card board sheets of dimension 25 cm by 40 cm to make container packets without top. Let x cm be the length of the side of the square to be cut out from each corner to give that sheet the shape of the container by folding up the flaps.
Based on the above information, answer the following equations.
(i) Express the volume (V) of each container as function of x only.
(ii) Find $\frac{dV}{dx}$.
(iii) For what value of x, the volume of each container is maximum?
(iv) Check whether V has a point of inflection at $x = \frac{65}{6}$ or not?

🕮 Unit IV - Vectors & 3 D Geometry

☑ Vector Algebra, Three Dimensional Geometry

Q01. A butterfly is moving in a straight path in the space.

Let this path be denoted by a line l whose equation is $\frac{x-1}{2} = \frac{2-y}{3} = \frac{z-3}{4}$ *say*.
Using the information given above, answer the following with reference to the line l.
(i) Write the position vector of the given point on the line.
(ii) What are the direction ratios of the line?
(iii) If the z-coordinate of a point on this line is 11, then write the x-coordinate of the same point on this line.
(iv) Write the vector equation of the given line.
(v) Write a unit vector in the direction of the vector parallel to the given line.

Q07. Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ respectively.

Based on the above information, answer the following questions.

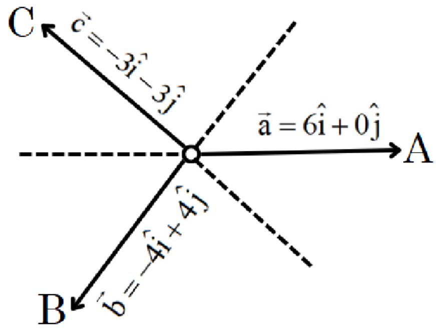
(i) Write the direction ratios of the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$.

(ii) Write a point, through which the line $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ passes.

(iii) Find the shortest distance between the given lines. Check if the lines intersect each other.

(iv) Will the lines intersect each other? Find the point at which the motorcycles may collide.

Q08. Teams A, B and C went for playing a tug of war game. Teams A, B and C have attached a rope to a metal ring and are trying to pull the ring into their own area.



Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}$ kN.

Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}$ kN.

Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}$ kN.

Based on the above information, answer the following.

(i) What is the magnitude of the force of Team A?

(ii) Which team will win the game?

(iii) Find the magnitude of the resultant force exerted by the teams.

(iv) In what direction is the ring getting pulled?

🕮 Unit VI - Probability

☑ Probability

Q01. The members of a consulting firm rent cars from three rental agencies :

Agency X

Agency Y

Agency Z

50% from agency X, 30% from agency Y and 20% from agency Z.
From past experience, it is known that 9% of the cars from agency X need a service and tuning before renting, 12% of cars from agency Y need a service and tuning before renting and 10% of the cars from agency Z need a service and tuning before renting.
Assume that the rental car delivered to the firm needs service and tuning.
For the information given above, answer the following.

(i) Find the probability that the cars need service and tuning, if it came from agency Y.
(ii) Find the probability that the cars need service and tuning, if it came from agency Z.
(iii) What is the probability that the car needs service and tuning?
(iv) If the rental car delivered to the firm need service and tuning, then find the probability that agency X is to be blamed.
(v) If the rental car delivered to the firm need service and tuning, then find the probability that agency Z is not to be blamed.

Q07. There are three categories of students in a class of 60 students :

A : Very hard working students

B : Regular but not so hard working

C : Careless and irregular.

It's known that 10 students are in category A, 30 in category B and rest in category C.
It is also found that probability of students of category A, unable to get good marks in the final year examination is, 0.002, of category B it is 0.02 and of category C, this probability is 0.20.

Based on the above information answer the following.

(i) If a student selected at random was found to be the one who could not get good marks in the examination, then find the probability that this student is of category C.
(ii) Assume that a student selected at random was found to be the one who could not get good marks in the examination. Then find the probability that this student is either of category A or of category B.
(iii) Find the probability that the student is unable to get good marks in the examination.
(iv) A student selected at random was found to be the one who could not get good marks in the examination. Then find the probability that this student is of category A.
(v) A student selected at random was found to be the one who could not get good marks in the examination. Then find the probability that this student is **NOT** of category A.

Q18. Three persons A, B and C apply for the job of Manager in a Private Company.

Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4.
The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively.

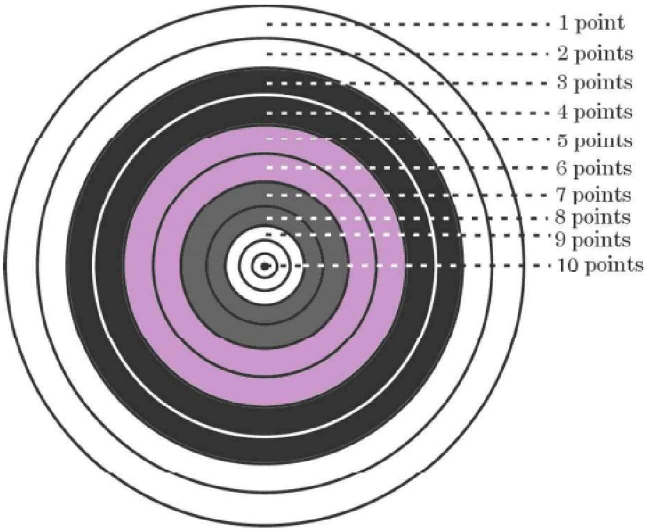
Based on the information given above, answer the following questions.

(i) If the change takes place in the company, then find the probability that it is due to the appointment of C.
(ii) If the change takes place in the company, then find the probability that it is due to the appointment of A.
(iii) If the change takes place in the company, then find the probability that it is due to the appointment of B.

(iv) Find the probability that the change takes place in the company.

(v) If the change does not take place, then determine the probability that it is due to the appointment of C.

Q27. In a game of Archery, each ring of the Archery target is valued. The centre-most ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.



Archer A is likely to earn 10 points with a probability of 0.8 and Archer B is likely the earn 10 points with a probability of 0.9.

Based on the above information, answer the following.

(i) Write the probability that archer A does not earn 10 points.

(ii) Write the probability that archer B does not earn 10 points.

(iii) If both of them hit the Archery target, then find the probability that exactly one of them earns 10 points.

(iv) If both of them hit the Archery target, then find the probability that both of them earn 10 points. Also, write the probability if none of them earns 10 point.

Q31. Read the following passage and the answer the questions given below.

A shopkeeper sells three types of flowers seeds A_1, A_2 and A_3.

These are sold as mixture, where their proportions are 4:4:2 respectively.

Also their germination rates are 45%, 60% and 35% respectively.

Let A_1 : seed A_1 is chosen, A_2 : seed A_2 is chosen and A_3 : seed A_3 is chosen 3.

Also let E : seed germinates.

(i) Find $P(A_1)$, $P(A_2)$ and $P(A_3)$.

(ii) Write $P(E|A_1)+P(E|A_2)+P(E|A_3)$.

(iii) Find the probability of a randomly chosen seed to germinate. Express your answer in %.

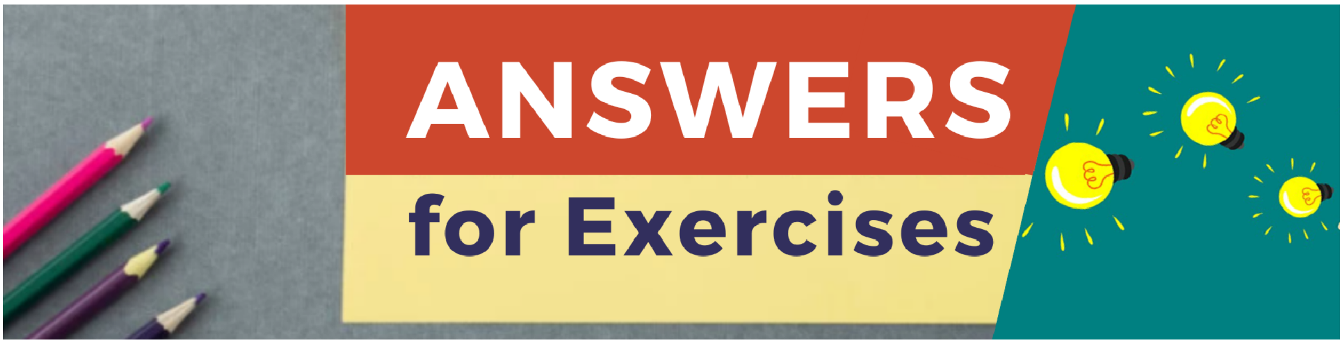
(iv) Calculate the probability that it is of the type A_2 given that a randomly chosen seed does not germinate.

Q38. Read the following passage and then answer the questions given below.

There are two anti craft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?



CHAPTER 01

EXERCISE 1.1

Q01. $1\times12, 2\times6, 3\times4, 4\times3, 6\times2, 12\times1$ Q02. (a) 2^6 Q01. (b) 2^9 Q01. (c) 81

Q03. (a) $\begin{bmatrix} 0 & -1/3 & -1/2 \\ 1/3 & 0 & -1/5 \\ 1/2 & 1/5 & 0 \\ 3/5 & 1/3 & 1/7 \end{bmatrix}$ Q03. (b) $\begin{bmatrix} \frac{1}{3} & 1 \\ 0 & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$ Q03. (c) $\begin{bmatrix} 0 & 5 & 8 \\ 0 & 0 & 7 \end{bmatrix}$

Q04. (a) 10λ Q04. (b) $\frac{1}{2}$ Q04. (c) $\frac{1}{2}$ Q04. (d) $e^{2x}\sin 2x$ Q05. 3

Q06. $\omega = 2n\pi, n \in Z$ Q07. $\begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$ Q08. $\text{diag}(11 \quad 9 \quad 2)$

Q09. I Q10. $X = \begin{bmatrix} -2 & -\frac{10}{3} \\ 4 & \frac{14}{3} \\ -\frac{31}{3} & -\frac{7}{3} \end{bmatrix}$ Q11. $\begin{bmatrix} 1 & -2 & 8 \\ 1 & -2 & -3 \end{bmatrix}$

Q12. (a) $x = 1, y = 2, z = 3, a = 4$ Q12. (b) $x = 2, y = 9$ Q12. (c) $x = 1, 2;\ y = 3 \pm 3\sqrt{2}$

Q12. (d) $a = -2, b = -7, c = -1, x = -3, y = -5, z = 2$ Q13. (a) 10

Q13. (b) $k = -4,\ a = -3$ Q13. (c) 11 Q13. (d) 0

EXERCISE 1.2

Q01. (a) $A = \frac{1}{5}\begin{bmatrix} 7 & -12 \\ -7 & 5 \end{bmatrix}, B = \frac{1}{5}\begin{bmatrix} -6 & 6 \\ 6 & 0 \end{bmatrix}$ Q01. (b) $\begin{bmatrix} 3/4 & 7/4 & -1/4 \\ -1 & -1 & 1/4 \\ 2 & 3/2 & -7/2 \end{bmatrix}$

Q02. (a) $x = 4, -3; y = -3, 4$ Q02. (b) $x = 1, y = 2$

EXERCISE 1.3

Q01. $a = 2, b = 3$ Q02. 1×1 Q03. $[18]$ Q04. $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Q05. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ Q06. I Q07. O Q08. (a) $x = 5, -3$

Q08. (b) $x = 2, y = 1$ Q08. (c) -1 Q08. (d) $x = 0$

Q09. 0 Q10. (a) I Q10. (b) A Q11. $\frac{1}{5}$

EXERCISE 1.4

Q01. $\begin{bmatrix}9\\12\\30\end{bmatrix}, \begin{bmatrix}1\\8\\-2\end{bmatrix}, \begin{bmatrix}10\\20\\28\end{bmatrix}$ Q02. $\begin{bmatrix}1 & -2 & -5\\3 & 4 & 0\end{bmatrix}$ Q03. $x=1,\ y=4$.

EXERCISE 1.5

Q01. $x = 2n\pi \pm \frac{\pi}{3},\ n \in Z$ Q03. (a) 5×2 Q03. (b) 3×4 Q04. (a) $p=0,\ q=3$

Q04. (b) $a=-\frac{2}{3},\ b=\frac{3}{2}$ Q08. Skew-symmetric Q10. $\begin{pmatrix}3 & 6\\6 & 9\end{pmatrix}$ Q11. $\begin{pmatrix}0 & 1/2 & -1\\-1/2 & 0 & -1/2\\1 & 1/2 & 0\end{pmatrix}$.

EXERCISE 1.6

Q02. $\begin{bmatrix}3 & 3 & 0\\6 & 4 & 2\end{bmatrix}$ Q03. $a=-2, b=-1$ Q06. $\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}, \begin{bmatrix}0 & a & b\\-a & 0 & c\\-b & -c & 0\end{bmatrix}$

Q07. $\begin{bmatrix}2 & \frac{3}{2}\\ \frac{3}{2} & 5\end{bmatrix} + \begin{bmatrix}0 & -\frac{5}{2}\\ \frac{5}{2} & 0\end{bmatrix}$ Q08. $\begin{bmatrix}2 & 11/2 & -5/2\\11/2 & 3 & 3/2\\-5/2 & 3/2 & 4\end{bmatrix} + \begin{bmatrix}0 & -3/2 & -7/2\\3/2 & 0 & 7/2\\7/2 & -7/2 & 0\end{bmatrix}$

EXERCISE 1.7

Q01. $\begin{pmatrix}2 & 0\\3 & 2\end{pmatrix}$ Q02. $A^{-1} = B = \begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$ Q03. $\begin{bmatrix}1 & -3/2\\0 & 1/2\end{bmatrix}$

Q04. $B^{-1} = \frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}$ Q05. $k=-1$.

EXERCISE 1.8

Q01. $x=\pm\frac{1}{\sqrt{2}}, y=\pm\frac{1}{\sqrt{6}}, z=\pm\frac{1}{\sqrt{3}}$ Q02. $\frac{1}{7}\begin{bmatrix}2 & 3\\-1 & -5\end{bmatrix}$

Q03. (a) $\begin{bmatrix}\frac{2}{5} & -\frac{1}{5} & 0\\ -\frac{1}{10} & \frac{3}{10} & 0\\ 0 & 0 & \frac{1}{5}\end{bmatrix}$ Q03. (b) $\frac{1}{4}\begin{pmatrix}3 & 1 & -1\\1 & 3 & 1\\-1 & 1 & 3\end{pmatrix}$ Q04. $\frac{1}{11}\begin{bmatrix}-3 & 4 & 5\\9 & -1 & -4\\5 & -3 & -1\end{bmatrix}$

Q05. $x=-4,\ y=1;\ A^{-1} = \begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix}$

EXERCISE 1.9

Q01. (a) $k=\frac{1}{19}$ Q02. $-12\ I$ Q03. $\begin{bmatrix}1 & -1 & -3\\-1 & -1 & -10\\-5 & 4 & 4\end{bmatrix}$

Q06. $\begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$ Q07. 6 I Q08. $\begin{pmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix}$

EXERCISE 1.10

Q01. The given information is expressed in matrix:

$$\begin{array}{l} \\ \\ \text{Appeared} \rightarrow \\ \text{Got through exam} \rightarrow \\ \text{Secured full marks} \rightarrow \end{array} \begin{array}{c} \text{School} \\ \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 25 & 35 \\ 20 & 20 \\ 15 & 10 \end{bmatrix} \end{array}$$

Q02. $7A = \begin{bmatrix} 56 & 112 \\ 224 & 336 \end{bmatrix}$. It represents the number of table fans and ceiling fans that the manufacturing units x and y produce in 7 days.

Q03. $\begin{bmatrix} \text{Teachers} \\ \text{Non-teaching staff} \\ \text{Principal} \\ \text{Vice Principal} \\ \text{Peon} \end{bmatrix} = \begin{bmatrix} 30 \\ 20 \\ 1 \\ 2 \\ 5 \end{bmatrix}$, Number of posts of each type $= \begin{bmatrix} 1500 \\ 1000 \\ 50 \\ 100 \\ 250 \end{bmatrix}$

Q04. $\begin{bmatrix} 130 & 250 & 190 \\ 140 & 310 & 230 \end{bmatrix}$ Q05. City C_1 : ₹7250; City C_2 : ₹6750

Q06. $\begin{bmatrix} 45 & 54 & 36 \\ 27 & 18 & 9 \end{bmatrix}$ Q07. Team A : ₹25250; Team B : ₹30900

Q08. Vehicle V_1 : ₹2040, Vehicle V_2 : ₹2240, Vehicle V_3 : ₹2310.

EXERCISE 1.11

Q01. (a) $a^2+b^2+c^2+d^2$ Q01. (b) 1 Q01. (c) pqr Q01. (d) 1

Q02. $(-3)^{1/3}$ Q03. (a) $\pm\frac{2}{\sqrt{3}}$ Q03. (b) 2 Q03. (c) ± 6 Q04. $\frac{\pi}{6}, \frac{\pi}{2}$ Q05. 0

Q06. $a_{13}M_{13} - a_{23}M_{23} + a_{33}M_{33}$ Q07. 2 Q08. (a) $2x - y = 0$

Q08. (c) $x = 12, -2$ Q09. (a) 0 Q09. (b) $\frac{1}{2}$ Q10. 0

Q11. (a) 0 Q11. (b) 0 Q11. (c) 0 Q11. (d) 0 Q11. (e) 0 Q11. (f) 0

Q11. (g) 0 Q12. (a) $-2(x^3+y^3)$ Q12. (b) $x^2(x+a+b+c)$ Q12. (c) $4xyz$

EXERCISE 1.12

Q01. [2, 4] OR $2(1+\sin^2\theta)$ Q02. (a) 0 Q02. (b) 0 Q02. (c) 1 Q02. (d) xy

Q05. –2 Q06. $x = -3,\ y = 4$ Q08. 1 Q09. 0

EXERCISE 1.14

Q01. $-\frac{7}{3}$ Q02. $-\frac{a}{3}$ Q03. 0, 1 Q04. $ax(3x+2a),\ 2ax+2a^2,\ 2a^2$

EXERCISE 1.15

Q01. (a) 40 Q01. (b) 5 Q01. (c) 25 Q01. (d) ± 8 Q01. (e) $\frac{1}{10}$ Q01. (f) 25 Q01. (g) $k^3|A|$

Q01. (h) $\pm\frac{9}{2}$ Q01. (i) $|A|^2$ Q01. (j) 27 Q01. (k) 25 Q01. (l) 162 Q02. (a) 3 Q02. (b) 17

Q02. (c) 2 Q02. (d) $\frac{\pi}{3}$ Q04. $\frac{1}{19}\begin{bmatrix}2 & 3\\5 & -2\end{bmatrix}$ Q05. $\begin{bmatrix}x^{-1} & 0 & 0\\0 & y^{-1} & 0\\0 & 0 & z^{-1}\end{bmatrix}$ Q06. $-14I_3$

Q07. –6 Q08. 2020

EXERCISE 1.16

Q01. (a) $\begin{bmatrix}2 & 0\\0 & -\frac{3}{2}\end{bmatrix}$ Q01. (b) $\begin{bmatrix}-191 & -110\\77 & 44\end{bmatrix}$ Q02. $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$ Q04. $\frac{1}{5}\begin{bmatrix}-4 & 1\\0 & 5\end{bmatrix}$

Q05. (b) $\begin{bmatrix}7 & -3 & -3\\-1 & 1 & 0\\-1 & 0 & 1\end{bmatrix}$ Q07. $\begin{bmatrix}-9 & -8 & -2\\8 & 7 & 2\\-5 & -4 & -1\end{bmatrix}$

Q08. (a) $\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}$ Q08. (b) $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ Q08. (c) $\begin{bmatrix}\frac{5}{3} & 2\\-\frac{2}{3} & 0\end{bmatrix}$

Q08. (d) $\begin{bmatrix}6 & 2\\\frac{11}{2} & 2\end{bmatrix}$ Q09. $\begin{pmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{pmatrix}$ Q10. $\begin{pmatrix}\cos\alpha & \sin\alpha & 0\\-\sin\alpha & \cos\alpha & 0\\0 & 0 & 1\end{pmatrix}$

EXERCISE 1.17

Q01. (a) $x=\frac{21}{5}, y=-\frac{1}{5}$ (b) $x=2, y=1, z=1$ (c) $x=2, y=1, z=3$

(d) $x=2, y=3, z=5$ (e) $x=y=z=1$

Q02. $A^{-1}=\frac{1}{62}\begin{bmatrix}-3 & 22 & 7\\5 & -16 & 9\\11 & 2 & -5\end{bmatrix}; x=\frac{96}{31}, y=\frac{40}{31}, z=\frac{24}{31}$

Q03. $A^{-1}=\frac{1}{14}\begin{bmatrix}8 & 2 & -2\\-7 & 0 & 7\\2 & 4 & -4\end{bmatrix}; x=2, y=-1, z=0$

Q04. $AB=2\,I;\ x=7,\ y=-10$ Q05. $AB=4I;\ x=1, y=2, z=-1$

Q06. $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}; x=0, y=5, z=3$ Q07. $x=\frac{5}{2},\ y=-1,\ z=-\frac{7}{2}$

Q08. (a) $x=\frac{5-3k}{2},\ y=k$ (b) $x=2+k, y=-k, z=k$.

EXERCISE 1.18

Q01. (a) ₹5, ₹8, ₹8 Q01. (b) $x+y+z=6, y+3z=11, x-2y+z=0; x=1, y=2, z=3$

Q02. Length = 200 m and breadth = 150 m

Q03. ₹300, ₹400, ₹500 For Tolerance, Kindness and Leadership, respectively.

Q04. 2000, 4400, 3600 (all in ₹) Q05. ₹880, ₹970, ₹500.

Q06. Let x and y be the initial investments by Mr. Nakul Saini in bond A and bond B respectively.
(i) $\begin{bmatrix} 2 & 3 \\ 8 & 11 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 80000 \\ 300000 \end{bmatrix}$ (ii) x = ₹10000, y = ₹20000.

Q07. ₹9900 in the City X and, ₹21200 in the City Y. Q08. 10%

Q09. Investment in 1st bond is ₹15000. And in 2nd bond, it's ₹20000. Q10. ₹25000

Q11. One English page : ₹10 and one Hindi page : ₹15 and, Charged from this poor boy : ₹65 less.

Q12. Amounts deposited : 1125, 1125, 4750 (in ₹) Q13. ₹7000, ₹6125, ₹7875; ₹21000

Q14. 24600 calories and 576 grams of proteins are needed for Family A and 15800 calories and 332 grams of proteins are needed for Family B.

Q15. Cost incurred by the organization for villages X, Y and Z respectively, are 30000, 23000 and 39000 (in ₹). Q16. ₹50500, ₹40800, ₹41600

Q17. ₹90,000 and ₹1,20,000. Q18. ₹200, ₹1000.

Q19. $(h, k) = (2, 3)$, $r = 5$. Q20. $a = -\frac{1}{2}$, $b = 8$, $c = 1$; $y = -\frac{1}{2}x^2 + 8x + 1$.

Q21. $a = 1$, $b = 3$, $c = 2$; $y = x^2 + 3x + 2$; $x = -1, -2$. Q22. ₹250, ₹300 and ₹775.

Q23. Orange juice : 2 litres, Beetroot juice : 3 litres, Kiwi juice : 1 litre.

CHAPTER 02

EXERCISE 2.1

Q01. No Q02. $R = \{(2,2),(2,4),(2,6),(2,8),(4,4),(4,8)\}$; $Dom.(R) = \{2,4\}$, $Range(R) = \{2,4,6,8\}$

Q03. $R = \{(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)\}$.
Domain for R is {1, 2, 3, 4, 6} and Range of R is {1, 2, 3, 4, 6}.

Q04. $Dom.(R) = \{\pm3, \pm2, \pm1, 0\}$, $Range(R) = \{0,1,2,3,4\}$.

Q05. Domain of R is $\{1,2,3,4,5,6\}$ and range of R is $\{5,6,7,8,9,10\}$.

Q06. $R = \{(1,1),(2,8),(3,27),(4,64)\}$, Domain of R is {1, 2, 3, 4} and Range of R is {1, 8, 27, 64}.

Q07. $R^{-1} = \{(-1,1),(0,2),(1,3),(3,5)\}$, Domain of R^{-1} is {–1, 0, 1, 3} and Range of R is {1, 2, 3, 5}.

Q08. Total 16 relations can be defined on A. These relations are as follows : $\phi, \{(1,1)\}, \{(1,2)\}, \{(2,1)\}$, $\{(2,2)\}, \{(1,1),(1,2)\}, \{(1,1),(2,1)\}, \{(1,1),(2,2)\}, \{(1,2),(2,1)\}, \{(1,2),(2,2)\}, \{(2,1),(2,2)\}$, $\{(1,1),(1,2),(2,1)\}, \{(1,1),(1,2),(2,2)\}, \{(1,1),(2,1),(2,2)\}, \{(1,2),(2,1),(2,2)\}$, $\{(1,1),(1,2),(2,1),(2,2)\}$.

EXERCISE 2.2

Q01. (a) {1, 2, 3} Q01. (b) {0, 2, 4} Q01. (c) R = {(1, 1), (2, 2), (3, 3)}

Q01. (d) {8, 27} Q01. (e) [1] = [3] = [5] = {1, 3, 5}, [2] = [4] = {2, 4}

Q01. (f) $[(2,3)] = \{(1,2), (2,3), (3,4)\}$

Q01. (g) $[3] = \{3,12,27,48\}$.

Q04. R is reflexive and transitive but not symmetric.

Q05. R is neither reflexive, symmetric nor transitive. Q06. R is not an equivalence relation.

Q07. R is neither reflexive, symmetric nor transitive.

Q08. R is reflexive and transitive but not symmetric. Q11. {1, 5, 9} Q20. {1}

Q21. R is reflexive but R is non symmetric and non transitive.

Q24. [(2, 5)] = {(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)} Q27. R is an equivalence relation.

Q29. R is reflexive, R is not symmetric, R is transitive.

Q31. Number of equivalence relations containing (1, 2) is two. Q32. One

Q33. $R_1 = \{(1,1),(2,2),(3,3)\}$, $R_2 = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$, $R_3 = \{(1,1),(2,2),(3,3),(1,3),(3,1)\}$,
$R_4 = \{(1,1),(2,2),(3,3),(2,3),(3,2)\}$, $R_5 = \{(1,1),(2,2),(3,3),(1,2),(2,1),(3,1),(1,3),(2,3),(3,2)\}$.

EXERCISE 2.3

Q01. $(2x+1)^2,\ 2x^2+1$ Q02. $||5x-2||,\ |5|x|-2|$ Q03. $\sin^2 x,\ \sin x^2$

Q04. (a) $x^4-6x^3+10x^2-3x$ Q04. (b) x

Q05. (a) $fog: R \to R,\ (fog)(x) = 4x^2-6x+1$ Q05. (b) $gof: R \to R,\ (gof)(x) = 2x^2+6x-1$

Q05. (c) $fof: R \to R,\ (fof)(x) = x^4+6x^3+14x^2+15x+5$

Q05. (d) $gog: R \to R,\ (gog)(x) = 4x-9$ Q06. $f(x) = \sin x$ and, $g(x) = x^2$.

Q07. (a) $1+\sin^2 x,\ \sin(x^2+1)$ Q07. (b) $fog = \{(2,5),(5,2),(1,5)\}$.

Q08. {(2, 7), (3, 7), (4, 11), (5, 11)}

Q09. Value of $fog = \begin{cases} 0, & \text{if } x \geq 0 \\ -4x, & \text{if } x < 0 \end{cases}$ and $gof = 0\ \forall x \in R$.

Q10. $gof = \{(-1,2),(0,0),(1,0),(2,2)\}$

EXERCISE 2.4

Q01. $\{-1, 1\}$ Q02. 3! i.e., 6.

Q03. (a) 9 Q03. (b) 8 Q03. (c) 60 Q03. (d) 0 Q03. (e) n!

Q04. **Horizontal Line Test:** The graph (b) represents the one-one function of x, because horizontal line drawn in (b) meets the graph at only one point.

Q05. **Vertical Line Test:** The graph of (a) represents the function of x, because vertical line drawn in (a) meets the graph at only one point i.e., for one x in domain there exists only one f (x) in codomain.

Q06. f is not one-one Q09. f is one-one Q10. f is not surjective

Q22. One-one and onto both Q32. Function is invertible Q37. Function f is not one-one

Q42. Not one-one.

EXERCISE 2.5

Q01. (a) One-one and onto both. Q01. (b) f is neither one-one nor onto.

Q01. (c) One-one and onto both. Q02. (a) $(-\infty,\infty)$

Q02. (b) [0, 1] Q02. (c) $R-\{0\}$ Q02. (d) $(3,\infty)$

Q02. (e) $[-2,0)\cup(0,1)$ Q03. (a) $\left[\frac{1}{3},1\right]$ Q03. (b) $(-\infty,0)\cup\left[\frac{1}{3},\infty\right)$

Q03. (c) $\{-1\}$ Q03. (d) $\{-1, 1\}$

Q04. (a) $(-\infty,-2)\cup[4,\infty)$ Q04. (b) $[1,2]$ Q05. $\left(-\infty,-\frac{8}{25}\right]\cup(0,\infty)$

Q06. $(-\infty,-1]\cup\left[\frac{1}{3},\infty\right)$.

CHAPTER 03

EXERCISE 3.1

Q01. a) $\frac{\pi}{6}$ b) $-\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{6}$ e) $\frac{3\pi}{4}$ f) $\frac{\pi}{4}$

g) $-\frac{\pi}{3}$ h) $\frac{2\pi}{3}$ i) $-\frac{\pi}{3}$ j) $\frac{\pi}{6}$ k) π l) $-\frac{\pi}{2}$

Q02. a) $-\frac{\pi}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{12}$ d) $-\frac{\pi}{3}$ e) $\frac{\pi}{3}$ f) $-\frac{\pi}{3}$

g) $\frac{3\pi}{4}$ h) 0 i) $-\frac{\pi}{6}$ j) $-\frac{\pi}{2}$

Q03. Below are the ranges of inverse trigonometric functions other than their principal branch :

1. For $\sin^{-1}x$, $\left[-\frac{3\pi}{2},-\frac{\pi}{2}\right]$, $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ etc.

2. For $\cos^{-1}x$, $[-\pi,0]$, $[\pi,2\pi]$ etc.

3. For $\operatorname{cosec}^{-1}x$, $\left[-\frac{3\pi}{2},-\frac{\pi}{2}\right]-\{-\pi\}$, $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]-\{\pi\}$ etc.

4. For $\sec^{-1}x$, $[-\pi,0]-\left\{-\frac{\pi}{2}\right\}$, $[\pi,2\pi]-\left\{\frac{3\pi}{2}\right\}$ etc.

5. For $\tan^{-1}x$, $\left(-\frac{3\pi}{2},-\frac{\pi}{2}\right)$, $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ etc.

6. For $\cot^{-1}x$, $(-\pi,0)$, $(\pi,2\pi)$ etc.

Q04. (a) $\left[-\frac{1}{2},\frac{1}{2}\right]$ Q04. (b) $|x|\le 1$ i.e., $x \in [-1, 1]$

Q04. (c) $x\in[-\sqrt{5},-\sqrt{3}]\cup[\sqrt{3},\sqrt{5}]$ Q04. (d) $x\in R\cap[-1,1]$ i.e., $[-1, 1]$

Q05. $[-1,1]$, $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$ Q09. Range $=\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$ Q10. Range $=\left[\frac{\pi}{2},\pi\right]$.

EXERCISE 3.2

Q01. a) π b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{6}$ e) $\frac{2\pi}{5}$

f) $\frac{\pi}{5}$ g) $\frac{\pi}{4}$ h) $-\frac{\pi}{6}$ i) $\frac{3\pi}{5}$ j) $\pi-2$

k) $3\pi-10$ l) $3-\pi$ m) 80° n) $-\frac{\pi}{6}$

Q02. a) 0 b) 0 c) $-\frac{7}{17}$ d) 1 e) $-\frac{24}{25}$

f) $\frac{\pi}{4}$ g) $\frac{5}{12}$ h) $\frac{3\pi}{4}$ i) $\frac{17}{6}$

Q03. $x\in\left(\frac{1}{\sqrt{2}},1\right]$ Q05. $-\frac{2\sqrt{6}}{5}$ Q06. $7-2\pi$ Q07. $\frac{\pi}{2}$

Q08. (a) $x\in(-\infty,1)$ (b) $x\in\left(-\frac{1}{4},0\right]$ (c) $x\in\left[1,\frac{7}{4}\right)$.

EXERCISE 3.3

Q02. a) $\sec^{-1}x$, if $x>1$; $\pi-\sec^{-1}x$, if $x<-1$ b) $-\frac{x}{2}$, if $-\pi<x<0$; $\frac{x}{2}$, if $0\le x<\pi$

c) $\frac{1}{2}\tan^{-1}x$ d) $x-\tan^{-1}\frac{4}{3}$ e) $\sin^{-1}\frac{x}{a}$

f) $\frac{\pi}{4}+\frac{x}{2}$, if $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\frac{x}{2}-\frac{3\pi}{4}$, if $x\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$

g) $\frac{\pi}{4}-x$ h) $\sin^{-1}x-\sin^{-1}\sqrt{x}$ i) $\frac{1}{2}\tan^{-1}x$ j) $3\tan^{-1}\frac{x}{a}$

k) $\frac{\pi}{4}+\frac{1}{2}\tan^{-1}x$ l) $\sqrt{1-x^2}$ m) $\frac{\pi}{4}+\sin^{-1}x$ n) $\frac{1}{2}\cos^{-1}\frac{x}{a}$

o) $\tan^{-1}\frac{x}{a}$ p) $\frac{\pi}{4}$

Q04 a) 0 b) $\frac{1}{5}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{1}{6}$ e) $-1, 0, 1$ f) 1

g) $\sqrt{3}$ h) $\frac{\sqrt{5}}{3}$ i) $\frac{-1\pm\sqrt{3}}{\sqrt{2}}$ j) 15 k) 4 l) $\pm\frac{2}{3}$

m) $\frac{1}{\sqrt{3}}$ n) $0, \frac{1}{2}$ o) 13 p) $-\frac{1}{2}$ q) $\frac{\sqrt{5}}{3}$

EXERCISE 3.4

Q02. 1 Q04. $\frac{3}{29}$ Q06. $\tan\frac{\pi}{12} = 2-\sqrt{3}$ Q09. $1-x^2$

Q11. $\sqrt{\frac{\sqrt{11}-3}{\sqrt{11}+3}}$ Q12. -1 Q14. $\left(\frac{1}{2}, 1\right)$

Q15. Least value $= \frac{\pi^2}{8}$, Greatest value $= \frac{5\pi^2}{4}$ Q16. $-\frac{1}{12}$

Q17. 0, –1 Q18. $\frac{1}{2\sqrt{2}}$.

CHAPTER 04

EXERCISE 4.1

Q01. Continuous Q02. Continuous Q03. Continuous everywhere in its domain

Q04. Continuous Q05. Continuous Q06. Continuous everywhere in its domain

Q07. Continuous everywhere in its domain $x \in (0, \infty) - 1$. Q08. $x \in R - \{0\}$ Q09. $x \in \left(0, \frac{1}{3}\right)$

Q10. $\frac{5}{3}$ Q11. Discontinuous Q12. Continuous

Q13. Point of discontinuity : $x = 1$ Q14. 3 Q15. $\frac{3}{4}$

Q16. 10 Q17. 4 Q18. $x = 0$ Q19. 1/2

EXERCISE 4.2

Q01. Continuous Q02. Continuous Q03. Continuous Q04. Discontinuous

Q05. Discontinuous Q06. Discontinuous Q07. 1 Q08. –4

Q09. $a = 3, b = -8$ Q10. $\frac{1}{2}$ Q11. $a = 1, b = -1$ Q12. $a = 3, b = 2$

Q13. $3a - 3b = 2$ Q14. $-\frac{2}{\pi}$ Q16. Discontinuous Q18. 9

Q19. $-\frac{2}{\pi}$ Q20. $a = 2, b = 1$

EXERCISE 4.3

Q02. $a = 3, b = -2$ Q03. $a = \frac{\pi}{6}, b = -\frac{\pi}{12}$ Q05. Yes Q06. No, 1

Q07. 8 Q08. Discontinuous Q09. $\frac{1}{2}$ Q13. Differentiable

Q14. Not Differentiable Q15. $a = 3, b = 5$ Q16. $p \in (0, 1]$ Q19. All integers (Z)

Q22. $p=-\frac{3}{2},\ q=\frac{1}{2}$ Q23. 20, 107 Q24. $p=-\frac{1}{2}$ Q26. $\frac{1}{4\sqrt{3}}$

Q27. $\frac{1}{2}$ Q28. $a=1, b=1\pm\sqrt{2};\ a=-1, b=1$ Q29. $a=4,\ b=-4$

Q30. Continuous at x = 0 & x = 1 and, non differentiable at x = 0 & x = 1

Q31. No point of discontinuity on [–1, 1] Q32. Continuous at x = 1 & non differentiable at x = 2

Q33. $\lambda=3$ and, non differentiable at x = 0 Q34. non differentiable at x = 1 & differentiable at x = 2

Q35. $p=\frac{1}{2},\ q=4$ Q36. $k=\frac{1}{2}$ Q38. $a=-1,\ b=4$ Q40. $a\neq 1,\ b=0,\ p=\frac{1}{3},\ q=-1$.

EXERCISE 4.4

Please note that the answers/ hints for this section are ***intentionally*** not given since you have studied this in your previous class in the Chapter 13 on Limits & Derivatives (Class XI).
Besides, if you wish to verify your answers, you may consult with your teacher.

EXERCISE 4.5

Q01. $\frac{3a}{a^2+x^2}$ Q02. $\frac{1}{\sqrt{1-x^2}}+\frac{2x}{\sqrt{1-x^4}}$ Q03. $\frac{1}{2\sqrt{x-x^2}}-\frac{1}{\sqrt{1-x^2}}$

Q04. $\frac{5a(a^2+6x^2)}{(a^2+9x^2)(a^2+4x^2)}$ Q05. $\frac{1-6x^2}{(1+9x^2)(1+4x^2)}$ Q06. $\frac{1}{2\sqrt{x}\,(1+x)}-\frac{1}{1+x^2}$ Q07. $\frac{1}{\sqrt{1-x^2}}$

Q08. $-\frac{1}{2\sqrt{1-x^2}}$ Q09. $\frac{1}{2\sqrt{1-x^2}}$ Q10. $\frac{2}{1+x^2}$

Q11. $-\frac{1}{2(1+x^2)}$ Q12. $\frac{1}{2\sqrt{1-x^2}}$ Q13. $-\frac{1}{1+x^2}$ Q14. $\frac{2}{\sqrt{1-x^2}}$

Q15. $-\frac{2}{\sqrt{1-x^2}}$ Q16. $-2x$ Q17. $\frac{2^{x+1}}{1+4^x}(\log 2)$ Q18. $\frac{1}{2}$

Q19. $-\frac{1}{2}$ Q20. $\frac{1}{2}$ Q21. $\frac{1}{\sqrt{1-x^2}}$ Q22. $\frac{1}{1+x^2}$

Q23. (a) $\frac{2}{1+x^2}$ Q23. (b) $\frac{2}{1+x^2}$ Q23. (c) $\frac{2}{1+x^2}$

Q24. $\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x}\sqrt{1-x}}$ Q25. $-\frac{x}{\sqrt{1-x^4}}$ Q26. $-\frac{2}{1+x^2}$.

EXERCISE 4.6

Q01. $\frac{1}{\sqrt{x^2+a^2}}$ Q02. $\sec x$ Q03. $\sec 2x$ Q04. $\frac{ab\cos x}{a^2-b^2\sin^2 x}$

Q05. $1-\sec x-\frac{2}{x}$ Q06. $2\sec 2x$ Q07. $\frac{2ab\cos x}{a^2-b^2\sin^2 x}$ Q08. $\frac{ab\sin x}{b^2\cos^2 x-a^2}$

Q09. $\left(\frac{x}{x\sin x+\cos x}\right)^2+\frac{1}{\sqrt{x^2-a^2}}$ Q10. $\frac{\operatorname{cosec} 2\sqrt{x+1}}{\sqrt{x+1}}+\frac{5x}{\sqrt[3]{1-x^2}}\left[\frac{3-x^2}{3x(1-x^2)}\right]+2\sin(4x-6)$

Q11. $\frac{4}{x^4+x^2+1}$ Q12. $\frac{2}{1+x^4}$

Q13. $\frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left\{\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}-\frac{1}{x-4}-\frac{1}{x-5}\right\}$

Q14. $\left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right]+x^{x+\frac{1}{x}}\left[\frac{x^2+1}{x^2}+\frac{x^2-1}{x^2}\log x\right]$

Q15. $x^{\cos^{-1}x}\left[\frac{\cos^{-1}x}{x}-\frac{\log x}{\sqrt{1-x^2}}\right]-\frac{1}{2\sqrt{x-x^2}}$

Q16. $x^{\sin x-\cos x}\left[\frac{\sin x-\cos x}{x}+(\cos x+\sin x)\log x\right]+\frac{x^x(1+\log x)+2\tan x\sec^2 x}{x^x+\sec^2 x}$

Q17. $\cos(x^x)\left[x^x(1+\log x)\right]+(\cos x)^x\left[\log\cos x-x\tan x\right]$

Q18. $2x^{\log x-1}\log x+(\log x)^x\left[\frac{1}{\log x}+\log\log x\right]$

Q19. $\frac{2}{1+x^2}-\frac{e^{\cos^{-1}x}}{\sqrt{1-x^2}}$

Q20. $x^{\tan x}\left[\frac{\tan x}{x}+\log x\ \sec^2 x\right]+\frac{4}{(e^x+e^{-x})^2}$

Q21. $(x\cos x)^x\left[1-x\tan x+\log(x\cos x)\right]+(x\sin x)^{1/x}\left[\frac{x\cot x+1-\log(x\sin x)}{x^2}\right]$

Q22. $\frac{x^x(1+\log x)+\sin x^{\cos x}(\cos x\cot x-\sin x\log\sin x)}{x^x+\sin x^{\cos x}}$

Q23. $e^{-ax^2}\cos(x\log x)(1+\log x)-2a\,xe^{-ax^2}\sin(x\log x)$

Q24. $\sqrt{x^2+a^2}$

Q25. $\sqrt{x^2-a^2}$

Q26. $\sqrt{a^2-x^2}$

Q27. $\frac{5(3-x)}{3(1-x)^{5/3}}+\frac{\log_{10}e}{x\left[1+(\log_{10}x)^2\right]}$

Q28. $x^{\cos^{-1}(x-1)}\left[\frac{\cos^{-1}(x-1)}{x}-\frac{\log x}{\sqrt{2x-x^2}}\right]+\frac{2}{(x+1)^2}$

Q29. (a) $-\frac{yx^{y-1}+y^x\log y}{xy^{x-1}+x^y\log x}$

Q29. (b) $-\frac{y^x\log y+y\,x^{y-1}+x^x(1+\log x)}{x\,y^{x-1}+x^y\log x}$

Q30. $\frac{y^{\cot x}\mathrm{cosec}^2 x\log y-y(\tan^{-1}x)^{y-1}(1+x^2)^{-1}}{(\tan^{-1}x)^y\log\tan^{-1}x+\cot x\ y^{\cot x-1}}$

Q31. 1

EXERCISE 4.7

Q01. $\frac{(x-y)y}{(x+y)x}$

Q02. $-\frac{3x^2+2xy+y\cos(xy)}{x^2+x\cos(xy)}$

Q03. $\frac{\sec^2(x+y)-\cos y-y\cos x}{\sin x-x\sin y-\sec^2(x+y)}$

Q04. $\frac{y}{x}$

Q05. $\frac{y^2}{2x(1-\log y)}$

Q06. $\frac{y^2\cot x}{1-y\log\sin x}$

Q07. $\frac{\sin x}{1-2y}$

Q10. $\frac{y-4x^3-4xy^2}{4x^2y+4y^3-x}$

EXERCISE 4.8

Q01. $-\frac{x}{y}$

Q02. 1

Q03. 1

Q04. $\tan\theta$

Q05. $\cot t$

Q06. $\frac{t(e^t-\sin t)}{1+t\cos t}$

Q07. $-\sqrt{\frac{1+t^2}{1-t^2}}$

Q08. 0

Q09. (a) $-\frac{(\sec t)(\mathrm{cosec}^2 t)}{a}$

Q09. (b) $\frac{8}{9a}$

Q10. $-4(5+3\sqrt{3})$

Q11. $\frac{1}{2}$

Q12. $4\sqrt{\frac{1-x^2}{1-4x^2}}$

Q13. $-\frac{1}{2}$

Q14. $\frac{1}{2}$

Q15. $-\frac{1}{4x(1+x^2)}$

Q16. $\frac{1}{2}$ Q17. 2 Q18. $\frac{1}{2}$ if $0 < x \le \frac{1}{\sqrt{2}}$; $-\frac{1}{2}$ if $\frac{1}{\sqrt{2}} \le x \le 1$

Q19. $\frac{1}{2}$, if we substitute $x = \sin\theta$; $-\frac{1}{2}$, if we substitute $x = \cos\theta$

Q20. $\frac{1}{4}$, if we substitute $x = \tan\theta$; $-\frac{1}{4}$, if we substitute $x = \cot\theta$ Q21. 4

Q22. $\frac{\sin x - x\cos x}{\sin^2 x\cos x}$ Q29. (a) $\frac{8\sqrt{2}}{a\pi}$ Q29. (b) $\frac{\sec^3 2t}{2at}$ Q30. 1

Q32. $-\frac{3}{2}$ Q34. 1 Q35. $-\frac{b}{a^2}$

EXERCISE 4.9

Q01. 0 Q02. $(x\ \mathit{log}\ x)^{-1}$ Q03. 0

Q05. It is true only for positive values of x because for log x to be defined, x must be greater than zero.

Q06. $\frac{1}{\sqrt{e}}$ Q07. $\frac{\pi}{180}\left[2\cos x^{\circ} - \frac{3}{2}\sin x^{\circ}\right]$ Q08. $-\cot x$

Q09. $f'(x) = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$ Q10. 3 Q11. 1 Q12. $\frac{2\mathit{ln}(\mathit{ln}\,x)}{x\,\mathit{ln}\,x}$ Q13. $2\cot x$

Q14. 0 Q15. 0 Q16. $\frac{\log_5 e}{x\log x}$ Q17. $5x^4$ Q18. 0 Q19. 0

Q20. $\frac{1}{4\sqrt{x(1-x)}\sin^{-1}\sqrt{x}}$ Q21. $\frac{\log_{10} e}{x}$ Q22. 5 Q24. -8.

EXERCISE 4.10

Q01. $\sec^2 y\ \tan y$ Q02. $\mathrm{cosec}\,x(\cot^2 x + \mathrm{cosec}^2 x)$ Q03. $\frac{1}{2}\mathrm{cosec}^2\frac{x}{2}\cot\frac{x}{2}$

Q17. $e^{\sin^2 x}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}\left[\sin 2x - \frac{1}{\sqrt{1-x^2}\cos^{-1}x}\right]$ Q38. $a^x\log_e a$ Q45. 0

Q66. $27\log 3 - 6$ Q67. $\sin\left(\frac{2x-1}{1+x^2}\right)^2 \times \frac{2(1-x^2+x)}{(1+x^2)^2}$ Q73. $x\left[2(1+\tan\log x) + \sec^2\log x\right]$

Q74. (a) $(x^x)^x.x[1+2\log x]$ Q74. (b) $x^{x^x}.x^x\left[\frac{1}{x} + \log x + (\log x)^2\right]$ Q76. 1 Q77. $\frac{2}{\sqrt{5}}$.

CHAPTER 05

EXERCISE 5.1

Q01. $1200\pi\ cm^2/s$ Q02. $\frac{7}{3}\ cm^2/s$ Q03. $\frac{\pi}{3}$ Q04. $\frac{27}{8}\pi(2x+1)^2$

Q05. $56\pi\ cm^2/s$ Q06. $64\ cm^2/min$, 16 cm/min Q07. $5\,cm^2/min, -2\,cm/min$

Q08. $\left(\frac{1}{48\pi}\right)$ cm/s Q09. (a) $\left(\frac{\sqrt{2}}{4\pi}\right)$ cm/s Q09. (b) $\frac{32}{27\pi}$ cm/s Q10. $\left(\frac{4}{9\pi}\right)$ cm/min

Q11. $\left(\frac{35}{88}\right)$ m/hr Q12. $\frac{147}{2}\pi\ cm^3/s$ Q14. $400\pi\ cm^3/cm$ Q15. $-48\ units^2/s$

Q16. (a) $\left(1,\frac{5}{3}\right),\left(-1,\frac{1}{3}\right)$ Q16. (b) $(2, 4)$ Q16. (c) $\left(-\frac{1}{2},-\frac{3}{4}\right)$

Q17. (a) 1 m/sec Q17. (b) 4 m/sec Q18. 2.5 km/hr Q19. $\left(\frac{8}{3}\right)$m/s

Q21. $b\sqrt{3}\,cm^2/s$ Q22. $t = 4sec,\ x = \frac{32}{3}m$ Q23. 8 cm/s^2 Q24. $2x^2 - 3x + 1$

Q26. 1 sec, 4 sec Q27. ₹18.315 Q28. ₹260 Q29. 30.255 Q30. 0

Q31. 112 Q32. 2 m/sec Q33. $\left(\frac{1}{\pi}\right)$ cm/sec Q34. $20\sqrt{3}\,cm^2s^{-1}$

Q35. $\left[\sqrt{2-\sqrt{2}}\right]$ v units/sec Q36. $-\frac{1}{20}$ radians/sec Q37. $-\frac{3}{\pi}$ m/min.

EXERCISE 5.2

Q01. Increasing : $(-\infty,-2]\cup[4,\infty)$; Decreasing : $[-2,4]$

Q02. Increasing : $[-2,-1]$; Decreasing : $(-\infty,-2]\cup[-1,\infty)$

Q03. Increasing : $[1,2]\cup[3,\infty)$; Decreasing : $(-\infty,1]\cup[2,3]$

Q04. Increasing : $(-\infty,2]\cup\left[\frac{12}{5},\infty\right)$; Decreasing : $\left[2,\frac{12}{5}\right]$

Q05. Increasing : $[1,\infty)$; Decreasing : $(-\infty,1]$

Q06. Increasing : $\left[0,\frac{3\pi}{4}\right]\cup\left[\frac{7\pi}{4},2\pi\right]$; Decreasing : $\left[\frac{3\pi}{4},\frac{7\pi}{4}\right]$

Q07. Increasing : $\left[0,\frac{\pi}{6}\right]$; Decreasing : $\left[\frac{\pi}{6},\frac{\pi}{2}\right]$

Q08. Increasing : $\left[\frac{\pi}{4},\frac{\pi}{2}\right]\cup\left[\frac{3\pi}{4},\pi\right]$; Decreasing : $\left[0,\frac{\pi}{4}\right]\cup\left[\frac{\pi}{2},\frac{3\pi}{4}\right]$ Q09. Increasing : $\left[0,\frac{\pi}{2}\right]$

Q10. Increasing : $(-\infty,-2]\cup[2,\infty)$; Decreasing : $[-2,2]-\{0\}$

Q11. Increasing : $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$; Decreasing : $\left[-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right]$

Q12. Increasing : $(2,3]$; Decreasing : $[3,\infty)$ Q13. Increasing : $(-1,\infty)$

Q14. Increasing : $[0,\infty)$; Decreasing : $(-1,0]$ Q15. Increasing : $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$; Decreasing : $\left[0,\frac{\pi}{4}\right]$

Q16. Increasing : $\left[0,\frac{\pi}{2}\right]\cup\left[\frac{3\pi}{2},2\pi\right]$; Decreasing : $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$

Q19. Increasing : $[e,\infty)$; Decreasing : $(0,e]-\{1\}$ Q20. $k\in\left(0,\frac{1}{3}\right)$ Q21. $a=-2$

Q22. $k\ge 2$ Q23. $a\ge 0$ Q27. Increasing : $\left[\frac{1}{e},\infty\right)$; Decreasing : $\left(0,\frac{1}{e}\right]$

Q28. Increasing : $(-\infty,-1]$; Decreasing : $[-1,\infty)$

Q32. Increasing : $(-2, 1)\cup(3,\infty)$; Decreasing : $(-\infty,-2)\cup(1, 3)$

Q33. Increasing : $[0,2]$; Decreasing : $(-\infty,0]\cup[2, \infty)$

Q35. Increasing : $\left[-\frac{1}{2},0\right)\cup\left[\frac{1}{2},\infty\right)$; Decreasing : $\left(-\infty,-\frac{1}{2}\right]\cup\left(0,\frac{1}{2}\right]$ Q36. $R-\{-1\}$

Q37. Strictly increasing in $\left(0,\frac{\pi}{4}\right)$ and $\left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$ and strictly decreasing in $\left(\frac{\pi}{4},\frac{7\pi}{12}\right)$ and $\left(\frac{11\pi}{12},\pi\right)$.

Q39. Increasing : $(0,\infty)$

Q40. Increasing : $\left[-\frac{1}{2},1\right]$

Q41. Yes, as $I'(x)>0\ \forall x\in R$

Q42. Neither strictly increasing nor strictly decreasing.

Q43. $x\in\left[\frac{1}{4},\infty\right)$

EXERCISE 5.3

Q01. a) Min. Value $=3$
b) Min. Value $=4$
c) Min. Value $=-1$
d) Max.Value $=\sin(1)$, Min. Value $=-\sin(1)$
e) Max.Value $=4$, Min. Value $=2$
f) Both do not exist
g) Min. Value $=24$
h) Max.Value $=2$
i) Max.Value $=4$, Min.Value $=2$
j) Max.Value $=3$, Min.Value $=$ doesn't exist

Q02. a) Local Max. at $x=\frac{\pi}{4}$, $f\left(\frac{\pi}{4}\right)=\sqrt{2}$

b) Local Max.at $x=\frac{3\pi}{4}$, $f\left(\frac{3\pi}{4}\right)=\sqrt{2}$; Local Min.at $x=\frac{7\pi}{4}$, $f\left(\frac{7\pi}{4}\right)=-\sqrt{2}$

c) Local Max.at $x=-1, f(-1)=0$; Local Min.at $x=-\frac{1}{5}, f\left(-\frac{1}{5}\right)=-\frac{3456}{3125}$,

Also, Point of Inflexion at $x=1$.

d) Local Max.at $x=\frac{2}{3}$, $f\left(\frac{2}{3}\right)=\frac{2\sqrt{3}}{9}$

e) Local Min.at $x=2$, $f(2)=2$

f) Local Min.at $x=-\frac{\pi}{4}$, $f\left(-\frac{\pi}{4}\right)=-\sqrt{2}$

g) Local Max.at $x=1$, $f(1)=-4$; Local Min.at $x=3$, $f(3)=-8$

h) Local Max.at $x=\frac{\pi}{6}$, $f\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}-\frac{\pi}{6}$; Local Min.at $x=-\frac{\pi}{6}$, $f\left(-\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{2}+\frac{\pi}{6}$

Q03. a) Absolute Max.$=\sqrt{2}$, Absolute Min.$=-1$
b) Absolute Max.$=\frac{5}{4}$, Absolute Min.$=1$
c) Absolute Max.$=2\pi$, Absolute Min.$=0$
d) Absolute Max.$=18$, Absolute Min.$=-\frac{9}{4}$
e) Absolute Max.$=8$, Absolute Min.$=-10$
f) Absolute Max.$=14\sqrt{2}$, Absolute Min.$=-\frac{2\sqrt{3}}{9}$

Q04. (a) Local Max.at $x=\frac{\pi}{6}$, $f\left(\frac{\pi}{6}\right)=\frac{3}{4}$; Local Min.at $x=\frac{\pi}{2}$, $f\left(\frac{\pi}{2}\right)=\frac{1}{2}$

(b) Local Min.at $x=\frac{\pi}{4}$, $f\left(\frac{\pi}{4}\right)=\frac{1}{2}$

(c) Local Max.at $x=\frac{\pi}{3}$, $f\left(\frac{\pi}{3}\right)=\sqrt{3}-\frac{\pi}{3}$; Local Min.at $x=-\frac{\pi}{3}$, $f\left(-\frac{\pi}{3}\right)=\frac{\pi}{3}-\sqrt{3}$

(d) Local Max. at $x=\frac{\pi}{6}$, $f\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}-\frac{\pi}{6}$; Local Min. at $x=-\frac{\pi}{6}$, $f\left(-\frac{\pi}{6}\right)=\frac{\pi}{6}-\frac{\sqrt{3}}{2}$

(e) Local Max. at $x=2$, $f(2)=\frac{1}{2}$

Q05. Local Max. at $x=0,-5$, $f(0)=105$, $f(-5)=\frac{295}{4}$; Local Min. at $x=-3$, $f(-3)=\frac{231}{4}$

Q06. $a=2,\ b=-\frac{1}{2}$

Q08. (a) Max. at $x=\frac{\pi}{6},\frac{5\pi}{6}$; Min. at $x=\frac{\pi}{2},\frac{3\pi}{2}$ Q08. (b) Max. Value $f(1)=e^{-1}$

Q09. (a) $a=-3,\ b=-9,\ c\in R$ Q09. (b) $m=2$ Q11. $a=120$.

Q12. Strictly increasing in $(-\infty,-2)$ and $\left(-\frac{4}{5},\infty\right)$ and strictly decreasing in $\left(-2,-\frac{4}{5}\right)$.

Also $x=-2$ is a point of local maximum and $x=-\frac{4}{5}$ is a point of local minimum.

EXERCISE 5.4

Q01. 8, 8 Q02. 420 Q03. 50 items Q07. $l=\sqrt{2}\,r,\ b=\frac{r}{\sqrt{2}}$, Area $=r^2$

Q11. $\frac{4R}{3}$ Q12. $2048\pi\ cm^3$ Q13. (a) (–2, –8)

Q13. (b) (4, –4) Q14. $\sqrt{\frac{4c-1}{2}}$ units Q15. $\frac{3\sqrt{2}}{8}$ units Q16. $\sqrt{5}$ units

Q17. (a) 4 cm, 1024 cm^3 Q17. (b) 5 cm Q18. $\left(\frac{h^2}{4}\right)$ sq.units

Q19. $(5+5\sqrt{2})$ m Q20. (a) $\frac{160}{3\sqrt{3}+4}$ cm, $\frac{120\sqrt{3}}{3\sqrt{3}+4}$ cm Q20. (b) $\frac{112}{\pi+4}$ cm, $\frac{28\pi}{\pi+4}$ cm

Q20. (c) $\frac{20\pi}{3\sqrt{3}+\pi}$ m, $\frac{60\sqrt{3}}{3\sqrt{3}+\pi}$ m Q20. (d) 16 cm, 18 cm; Area : 34 cm^2

Q21. $\left(\frac{5}{2}\right)$ cm each Q23. $\theta=\frac{\pi}{2}$, Area $=\left(\frac{ab}{2}\right)$ sq.units Q26. $\left(\frac{1000}{27\pi}\right)m^3$

Q27. $r=\sqrt[3]{\frac{100}{\pi}}$, $h=\sqrt[3]{\frac{100}{\pi}}$ Q29. (b) Length of square base = 3 cm and height of the box = 1.5 cm.

Q30. $\frac{4\pi R^3}{3\sqrt{3}}$ units^3 Q33. $75\sqrt{3}\ cm^2$ Q34. $r=\left(\frac{50}{\pi}\right)^{1/3}$, $h=2r$

Q38. Maximum profit = Rs.76/- when x = 240. For charity = Rs.7.60/-

Q39. 7 cm Q41. Radius = 4 cm and height = 8 cm. Q42. $\frac{2x^3}{3}\left(1+\frac{2\pi}{27}\right)$ cubic units

Q43. Four hours per day. Q46. Length: 15 cm, Breadth: 10 cm.

Q47. $2c\sqrt{ab}$ Q49. 10 m Q50. (a) $\frac{3\sqrt{3}\,ab}{4}$ units^2

Q50. (b) 2ab units^2 Q52. 2 : 1 Q53. (a) $l:\frac{20}{\pi+4}$ m, $b:\frac{10}{\pi+4}$ m

Q53. (b) $l : \frac{12}{6-\sqrt{3}}\text{m}, b : \frac{18-6\sqrt{3}}{6-\sqrt{3}}\text{m}$ Q55. $t = \frac{1}{100\pi}\left(\frac{\pi}{2} - 0.04\right)$, $I = 50$

Q56. (b) $\frac{1}{2}x^2$ sq.units Q58. $100x - x^2$, $6x^2 - \frac{x^3}{3} - 11x - 50$; $x = 11$, Max. Profit $= \frac{334}{3}$

Q59. (a) Rs.1000/- Q59. (b) Rs.1920/- Q61. Rs.100/-.

CHAPTER 06

EXERCISE 6.1

☑ Type A

Q01. $\log|x| + \frac{x^2}{2} - 3$ Q02. $\tan x - 16x - 9\cot x + C$ Q03. $-\frac{1}{x} + 2x + \frac{x^3}{3} + C$ Q04. $\frac{\pi x}{2} - \frac{x^2}{2} + C$

Q05. $\frac{2}{5}x^{5/2} - 4x^{3/2} + 18\sqrt{x} + k$ Q06. $\frac{x^2}{2} + 2x + \log|x| + k$

Q07. $\frac{2}{3}\left\{(x+3)^{3/2} - (x+2)^{3/2}\right\} + k$ Q08. $\frac{x^3}{3} + x + \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + k$ Q09. $\frac{x^3}{3} + k$

Q10. $\frac{a^{3x+3}}{3\log a} + k$ Q11. $\frac{180}{\pi}\sec x^\circ + k$ Q12. $\sec x + \text{cosec } x + k$

Q13. $\log|1 - \cos x| + k$ Q04. $2\cos x - 2x\sin\alpha + k$ Q15. $2\sin x + x + k$

Q16. $-\frac{11}{2}x - \frac{1}{4}\sin 2x - 9\cot x + k$ Q17. $2\tan x - 3\sec x + k$ Q18. $\tan x - \tan^{-1} x + k$

Q19. $\frac{\pi}{4}x - \frac{x^2}{4} + k$ Q20. $\frac{\pi x}{2} - \frac{x^2}{4} + k$ Q21. $\frac{\pi}{4}x + \frac{x^2}{4} + k$ Q22. $\sin^{-1} x + k$

Q23. $\alpha = \frac{\pi}{4}$, $\beta =$ Integral constant Q24. $p = -\frac{\pi}{4}$, $q =$ Integral constant Q25. $6x + C$.

EXERCISE 6.2

☑ Type B

Q01. $\frac{1}{2}(2x-1)^{5/2} + \frac{11}{6}(2x-1)^{3/2} + k$ Q02. $\frac{1}{27}(30x - 176)\sqrt{3x+7} + k$ Q03. $\log|x+1| - \frac{1}{x+1} + k$

Q04. $\frac{1}{8}\cos(7 - 4x^2) + k$ Q05. $\frac{1}{2}\sqrt{4 - (\log x)^2}\,\log x + 2\sin^{-1}\left(\frac{\log x}{2}\right) + k$

Q06. $-\frac{2}{3}\log\left|1 - 3\sqrt{x}\right| + k$ Q07. $2\log\left|\sec e^{\sqrt{x}} + \tan e^{\sqrt{x}}\right| + k$ Q08. $\log\log\log x + k$

Q09. $\frac{1}{6}\left(e^{\log x} + \log x\right)^6 + k$ Q10. $-\frac{1}{2}\cos\left(1 + (\log x)^2\right) + k$ Q11. $-\frac{1}{2}\left[\log\left(1 + \frac{1}{x}\right)\right]^2 + k$

Q12. $\log\left|x + \log\tan\frac{x}{2}\right| + k$ Q13. $\frac{1}{(\log 2)^3}.2^{2^{2^x}} + k$ Q14. $\frac{1}{3}e^{\tan^{-1}3x} + k$

Q15. $-\frac{1}{3}\sin\left(\cot^{-1}x^3\right) + k$ Q16. $\frac{1}{\log 3}(3^{x+\cot^{-1}x}) + k$ Q17. $-\frac{1}{\sin x + \cos x} + k$

Q18. $\frac{1}{2(a^2 - b^2)}\log\left|a^2\sin^2 x + b^2\cos^2 x\right| + k$ Q19. $-\frac{1}{4}\sin^{-1}\left(\frac{\cos^2 2x}{3}\right) + k$ Q20. $2\sqrt{x + \cos^2 x} + k$

Q21. $\log|x - \log\cos x| + k$ Q22. $-2\sqrt{\cot x} + k$ Q23. $\sqrt{\tan x^2} + C$

Q24. $\frac{1}{3}\sin e^{x^3}+k$ Q25. $-\left(\cot(xe^x)+xe^x\right)+k$ Q26. $\cot\left(\frac{1}{x}\right)+k$

Q27. $\frac{2}{11}(\sin x)^{11/2}-\frac{4}{7}(\sin x)^{7/2}+\frac{2}{3}(\sin x)^{3/2}+k$ Q28. $\frac{1}{9}(1+\sin 6x)^{3/2}+k$

Q29. $\frac{2}{5}\tan^{5/2}x+k$ Q30. $2\left[\sqrt{e^x-1}-\tan^{-1}\sqrt{e^x-1}\right]+k$ Q31. $\sec^{-1}x+k$

Q32. $\frac{2}{9}(1+x^3)^{3/2}-\frac{2}{3}(1+x^3)^{1/2}+k$ Q33. $\log|x|-\frac{2}{n}\log\left(1+\sqrt{1+x^n}\right)+k$ Q34. $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2}+C$

Q35. $3\log\left|x^{1/3}+\sqrt{x^{2/3}-4}\right|+k$ Q36. $\left(1-\frac{1}{x^4}\right)^{1/4}+k$ Q37. $-\frac{1}{2}\left(1+\frac{1}{x^5}\right)^{2/5}+C$

Q38. $\frac{4}{15}\left(1-\frac{1}{x^3}\right)^{5/4}+k$ Q39. $-\frac{2}{a\sqrt{a}}\left[(a-1)\log\left|1-\sqrt{ax}\right|+(2-a)\left(1-\sqrt{ax}\right)-\frac{1}{2}\left(1-\sqrt{ax}\right)^2\right]+k$

Q40. $\frac{1}{a^2}\left(\frac{x}{\sqrt{a^2-x^2}}\right)+k$ Q41. $\frac{1}{2}\sin^2(x^2+1)+k$ Q42. $-2\sqrt{9-\sin^{-1}x}+k$

Q43. $\frac{1}{e}\log\left|x^e+e^x\right|+k$ Q44. $-\frac{(1-x)^{24}(1+24x)}{600}+k$ Q45. $\log|\cos x+x\sin x|+C$

Q46. $-\frac{1}{2(1-\cos x)^2}+k$ Q47. $\frac{x^{2023x}}{2023}+C$ Q48. $\frac{1}{3}x^{3x}-x^x+C$

Q49. $\frac{1}{2}\left(\frac{\left\{x+\sqrt{1+x^2}\right\}^{n+1}}{n+1}+\frac{\left\{x+\sqrt{1+x^2}\right\}^{n-1}}{n-1}\right)+C$ Q50. $-\frac{2}{b^2}\left[\log|a+b\cos x|+\frac{a}{a+b\cos x}\right]+k$

Q51. $\log\left|\frac{x}{x+\sin x}\right|+C$ Q52. $-\frac{2}{3}\sin^{-1}(\cos^{3/2}x)+k$ Q53. $\log|\sin x|-\sin x+k$

☑ Type C

Q01. $\sin^{-1}\left(\frac{x+3}{4}\right)+k$ Q02. $\frac{1}{\sqrt{3}}\log\left|x-\frac{13}{6}+\sqrt{\left(x-\frac{13}{6}\right)^2-\frac{49}{36}}\right|+k$ Q03. $\frac{1}{\sqrt{2}}\sin^{-1}(4x-1)+k$

Q04. $\sin^{-1}\left(\frac{x-a}{a}\right)+k$ Q05. $\log\left|x-\frac{a+b}{2}+\sqrt{(x-a)(x-b)}\right|+k$ Q06. $\frac{1}{5}\log\left|\frac{3+2x}{1-x}\right|+k$

Q07. $\frac{1}{2}\tan^{-1}(2e^x)+k$ Q08. $\frac{1}{4}\log\left|\frac{e^x+1}{e^x+5}\right|+k$ Q09. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2-1}{\sqrt{3}}\right)+k$

Q10. $\log\left|\frac{2\log x+1}{3\log x+2}\right|+k$ Q11. $\log\left|\sin x-1+\sqrt{\sin^2 x-2\sin x-3}\right|+k$

Q12. $\log\left|\log x+\sqrt{(\log x)^2-9}\right|+k$ Q13. $\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+k$ Q14. $\frac{1}{48}\log\left|\frac{4-3\cos^2 x}{4+3\cos^2 x}\right|+k$

Q15. $-\frac{2}{\sqrt{39}}\tan^{-1}\left(\frac{2\cot x+1}{\sqrt{39}}\right)+k$ Q16. $\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+\sin x}{\sqrt{2}-\sin x}\right|+k$ Q17. $3\log|x+1|+\frac{5}{x+1}+k$

Q18. $\frac{1}{8}\log|x|+\frac{7}{8}\log|5x-8|+k$ Q19. $2\sqrt{x^2-4x+5}+7\log\left|x-2+\sqrt{x^2-4x+5}\right|+k$

Q20. $2\sin^{-1}\left(\frac{2x-1}{5}\right)-2\sqrt{6+x-x^2}+k$

Q21. $\sqrt{ax+x^2}+\frac{a}{2}\log\left|x+\frac{a}{2}+\sqrt{x^2+ax}\right|+C$ OR $x=a\tan^2\theta \Rightarrow \sqrt{ax+x^2}+a\log\left|\sqrt{a+x}+\sqrt{x}\right|+k$

Q22. $\sqrt{x+x^2}+\frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{x+x^2}\right|+k$

Q23. $\sqrt{1+x^2}+\log\left|\sqrt{1+\frac{1}{x^2}}-\frac{1}{x}\right|+k$

Q24. $8\sin^{-1}\left(\frac{x^2}{16}\right)+\frac{1}{2}\sqrt{256-x^4}+k$

Q25. $\sin^{-1}x+\sqrt{1-x^2}+k$

Q26. $\frac{1}{5}\left[\sin^{-1}5x+\sqrt{1-25x^2}\right]+k$

Q27. $\frac{1}{2}(x+2)\sqrt{x^2+4x+8}+2\log\left|(x+2)+\sqrt{x^2+4x+8}\right|+k$

Q28. $\frac{x-6}{2}\sqrt{(x-5)(7-x)}+\frac{1}{2}\sin^{-1}(x-6)+k$

Q29. $\sqrt{2}\left[\frac{2x-7}{4}\sqrt{7x-10-x^2}+\frac{9}{8}\sin^{-1}\left(\frac{2x-7}{3}\right)\right]+k$

Q30. $\frac{1}{2}(x-2a)\sqrt{4ax-x^2}+2a^2\sin^{-1}\left(\frac{x-2a}{2a}\right)+k$

Q31. $\frac{1}{8}(2x^2-3)\sqrt{x^4-3x^2+1}-\frac{5}{16}\log\left|x^2-\frac{3}{2}+\sqrt{x^4-3x^2+1}\right|+k$

Q32. $-\frac{1}{3}(1+x-x^2)^{3/2}+\frac{1}{2}\left[\frac{1}{2}\left(x-\frac{1}{2}\right)\sqrt{1+x-x^2}+\frac{5}{8}\sin^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)\right]+k$

Q33. $\frac{2}{3}(x^2+4x+3)^{3/2}-\frac{1}{2}\left[(x+2)\sqrt{x^2+4x+3}-\log\left|x+2+\sqrt{x^2+4x+3}\right|\right]+k$

Q34. $\frac{4}{3}[\sin^2x-4\sin x+5]^{\frac{3}{2}}+\frac{7}{2}\left[(\sin x-2)\sqrt{\sin^2x-4\sin x+5}+\log\left|\sin x-2+\sqrt{\sin^2x-4\sin x+5}\right|\right]+C$

Q35. $\frac{3}{2}\sin^{-1}x-\frac{x}{2}\sqrt{1-x^2}+3\sqrt{1-x^2}+C$.

EXERCISE 6.3

☑ **Type D**

Q01. $\frac{\sin 4x}{8}-\frac{\sin 10x}{20}+k$

Q02. $\frac{\cos x}{2}-\frac{\cos 3x}{6}+k$

Q03. $\frac{1}{4}\left[x+\frac{\sin 6x}{6}+\frac{\sin 4x}{4}+\frac{\sin 2x}{2}\right]+k$

Q04. $\frac{\cos 6x}{24}-\frac{\cos 4x}{16}-\frac{\cos 2x}{8}+k$

Q05. $\frac{2}{3}\sin 3x+2\sin x+k$

Q06. $\frac{1}{3}\log|\sec 3x|-\frac{1}{2}\log|\sec 2x|-\log|\sec x|+k$

Q07. $\frac{1}{2}\log|\sec 2x|-\log|\sec(x-\theta)|-\log|\sec(x+\theta)|+k$

Q08. $\frac{3}{8}x+\frac{\sin 4x}{32}+\frac{\sin 2x}{4}+k$

Q09. $\sin 2x-x+C$

Q10. $\frac{1}{128}\left[3x-\sin 4x+\frac{\sin 8x}{8}\right]+k$

Q11. $-\frac{1}{3}\cot^3x+\cot x+x+k$

Q12. $\frac{1}{3}\tan^3x-\tan x+x+k$

Q13. $\frac{1}{3}\tan^3x+\tan x+k$

Q14. $\frac{1}{5}\tan^5x+\frac{2}{3}\tan^3x+\tan x+k$

Q15. $\frac{1}{3}\cos^3x-\cos x+k$ OR $\frac{\cos 3x}{12}-\frac{3\cos x}{4}+k$

Q16. $-\frac{1}{5}\cos^5x+\frac{2}{3}\cos^3x-\cos x+k$

Q17. $\frac{1}{64}\left[\frac{\cos 6x}{3}-3\cos 2x\right]+k$ OR $\frac{1}{16}\left[\frac{\cos^3 2x}{3}-\cos 2x\right]+k$

Q18. $\frac{1}{2}\tan^2 x - \log|\sec x| + k$ Q19. $x\cos a + \sin a \log|\cos(x-a)| + k$

Q20. $x\cos a - \sin a \log|\sin x| + k$ Q21. $x\cos(b-a) - \sin(b-a)\log|\sin(x+b)| + k$

Q22. $x\cos 2a - \sin 2a \log|\sin(x+a)| + k$ Q23. $\frac{1}{3}\log|\sin 3x| - \frac{1}{5}\log|\sin 5x| + k$

Q24. $\frac{1}{\cos(a-b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right| + k$ Q25. $\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + k$

Q26. $\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + k$ Q27. $\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x+b)}{\cos(x+a)}\right| + k$

Q28. $\tan^{-1}\left(1+\tan\frac{x}{2}\right) + k$ Q29. $\frac{1}{2\sqrt{5}}\log\left|\frac{\sqrt{5}\tan x - 1}{\sqrt{5}\tan x + 1}\right| + k$ Q30. $\frac{1}{5}\log\left|\frac{3\tan\frac{x}{2} - 1}{\tan\frac{x}{2} + 3}\right| + k$

Q31. $\frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)\right| + k$ OR $\frac{1}{2}\log\left|\frac{1+\sqrt{3}\tan\frac{x}{2}}{\sqrt{3}-\tan\frac{x}{2}}\right| + k$ Q32. $\frac{1}{\sqrt{2}}\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right| + k$

Q33. $-\sqrt{2}\log\left|\sec\left(\frac{\pi}{4}-\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right| + k$ Q34. $2\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right) + k$ Q35. $-2\left[\cos\frac{x}{2}+\sin\frac{x}{2}\right] + k$

Q36. $2\tan\frac{x}{2} - x + k$ Q37. $2(\operatorname{cosec} x - \cot x) - x + k$ Q38. $2(\sec x + \tan x) - x + k$

Q39. $\log|\sin x + \cos x| + k$ Q40. $\frac{x}{2} + \frac{1}{4}\log|\cos 2x + \sin 2x| + k$

Q41. $\frac{18}{25}x + \frac{1}{25}\log|3\sin x + 4\cos x| + k$ Q42. $\frac{3}{25}x - \frac{4}{25}\log|4\cos x + 3\sin x| + k$

Q43. $\frac{4}{25}x + \frac{3}{25}\log|4\cos x + 3\sin x| + k$ Q44. $\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right) + k$

Q45. $\frac{1}{2}\log\left|\frac{\tan x}{\tan x + 2}\right| + k$ Q46. $\frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x) + k$ Q47. $\frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + k$

Q48. $\frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right) + k$ Q49. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right| + k$ Q50. $\frac{1}{4\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan 2\theta}{\sqrt{3}-\tan 2\theta}\right| + C$

Q51. $-\frac{2\sqrt{\cos a + \sin a \cot x}}{\sin a} + k$ Q52. $-\frac{2\sqrt{\cos a - \sin a \tan x}}{\sin a} + k$

EXERCISE 6.4

☑ Type E

Q01. $\frac{3}{2}\log|x-1| + \frac{7}{2}\log|x-3| - 5\log|x-2| + k$ Q02. $x - \frac{1}{4}\log|x+1| + \frac{9}{4}\log|x-3| + k$

Q03. $\frac{x}{3} + \log|x| - \frac{8}{9}\log|1-3x| + k$ Q04. $x - 2\log|x| + \frac{3}{2}\log|x-1| + \frac{1}{2}\log|x+1| + k$

Q05. $\frac{5}{6}\log|x+1| - \frac{1}{14}\log|x-1| - \frac{16}{21}\log|2x+5| + k$ Q06. $\log|x+1| - 5\log\left|\frac{x+2}{x+3}\right| + k$

Q07. $-\frac{1}{2}\log|x+1|+\frac{1}{4}\log|x^2+9|+\frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right)+C$

Q08. $\frac{2}{3}\log|x-1|-\frac{1}{3}\log|x^2+x+1|+\frac{2}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}}+k$

Q09. $\frac{4}{5}\log|x+4|+\frac{1}{5}\log|x-1|-\frac{1}{x-1}+k$

Q10. $\frac{1}{2}\log\left|\frac{x+1}{x-1}\right|-\frac{4}{x-1}+k$

Q11. $\log|x-1|-\frac{3}{x-1}-\frac{3}{2(x-1)^2}+k$

Q12. $x+\frac{x^2}{2}+\frac{1}{2}\log|x-1|-\frac{1}{4}\log|x^2+1|-\frac{1}{2}\tan^{-1}x+k$

Q13. $\frac{x^2}{2}+3x-\log|x-1|+8\log|x-2|+k$

Q14. $x+\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+k$

Q15. $\frac{1}{3\sqrt{2}}\left[\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\tan^{-1}(\sqrt{2}\,x)\right]+k$

Q16. $\frac{2}{3}\tan^{-1}\left(\frac{x}{2}\right)-\frac{1}{3}\tan^{-1}x+k$

Q17. $x-\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{3}{4}\tan^{-1}\left(\frac{x}{2}\right)+k$

Q18. $\log|\operatorname{cosec}x-\cot x|-2[\tan x-\sec x]+k$

Q19. $\log|e^{-x}+1|-2\log|e^{-x}+2|+k$

Q20. $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+k$

Q21. $\frac{1}{4}\log\left|\frac{x^4}{1+x^4}\right|+k$

Q22. $-\frac{1}{20}\log\left|\frac{x^5}{x^5-4}\right|+k$

Q23. $\frac{1}{27}\log\left|\frac{2+\sin x}{1-\sin x}\right|+\frac{1}{9(1-\sin x)}+\frac{1}{6(1-\sin x)^2}+k$

Q24. $\frac{1}{6}\log|\tan^2 x-\tan x+1|-\frac{1}{3}\log|\tan x+1|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+k$

Q25. $-\frac{1}{3}\log|\tan x+1|+\frac{1}{6}\log|\tan^2 x-\tan x+1|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$

Q26. $\frac{1}{3}\log\left|\frac{1+2\sec x}{2+\sec x}\right|+k$

Q27. $\frac{1}{2}\log|1-\cot^2 x|+k$

Q28. $\log\left|\frac{2-\sin x}{1-\sin x}\right|+k$

Q29. $\frac{2}{3}\log\left|\frac{1+2\log x}{2+\log x}\right|+k$

Q30. $\frac{1}{4}\log\log x-\frac{1}{8}\log|4+(\log x)^2|+k$

Q31. $2\log|6-\cos^2 x-4\sin x|+7\tan^{-1}(\sin x-2)+k$

Q32. $3\log|2-\sin\theta|+\frac{4}{2-\sin\theta}+k$

Q33. $\log\left|\frac{2+\tan x}{3+\tan x}\right|+k$

Q34. $\log\left|\frac{x^2+1}{x^2+2}\right|+k$

Q35. $\frac{1}{4}\log\left|\frac{1+\sin x}{1-\sin x}\right|+\frac{1}{2(1+\sin x)}+k$

Q36. $\frac{1}{10}\log(1-\cos x)-\frac{1}{2}\log(1+\cos x)+\frac{2}{5}\log(3+2\cos x)+k$

Q37. $\frac{1}{6}\log|1-\cos x|+\frac{1}{2}\log|1+\cos x|-\frac{2}{3}\log|1+2\cos x|+C$

Q38. $-\log|1-\sin x|+\frac{1}{2}\log|1+\sin^2 x|+\tan^{-1}\sin x+C$

EXERCISE 6.5

☑ Type F

Q01. $\sin x - x\cos x + k$

Q02. $2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + k$

Q03. $(x^2 - 2x + 2)e^x + k$

Q04. $\frac{e^{ax}}{a^2 + b^2}\left[a\cos bx + b\sin bx\right] + k$

Q05. $\frac{e^{-x}}{2}\left[\sin x - \cos x\right] + k$

Q06. $\frac{e^{2x}}{5}\left[2\sin x - \cos x\right] + k$

Q07. $\frac{1}{6}e^{x^3}(\sin x^3 + \cos x^3) + k$

Q08. $x\cot^{-1}x + \frac{1}{2}\log(1 + x^2) + k$

Q09. $\frac{1}{4}\left[(2x^2 - 1)\sin^{-1}x + x\sqrt{1 - x^2}\right] + k$

Q10. $\frac{1}{2}\left[(1 + x^2)\tan^{-1}x - x\right] + k$

Q11. $\frac{x^2}{2}\cot^{-1}x - \frac{1}{2}\tan^{-1}x + \frac{x}{2} + k$

Q12. $\frac{1}{2}\sqrt{x}\sqrt{1 - x} - \frac{1}{2}(1 - 2x)\sin^{-1}\sqrt{x} + k$

Q13. $x\sec^{-1}\sqrt{x} - \sqrt{x - 1} + k$

Q14. $\log\left|\frac{1 - \sqrt{1 - x^2}}{x}\right| - \frac{\sin^{-1}x}{x} + k$

Q15. $\frac{x\sin^{-1}x}{\sqrt{1 - x^2}} + \frac{1}{2}\log\left|1 - x^2\right| + k$

Q16. $\sqrt{1 + x^2}\tan^{-1}x - \log\left|x + \sqrt{1 + x^2}\right| + k$

Q17. $x - \sqrt{1 - x^2}\sin^{-1}x + k$

Q18. $-[\sqrt{1 - x^2}\cos^{-1}x + x] + C$

Q19. $x(\sin^{-1}x)^2 - 2\left[x - \sqrt{1 - x^2}\sin^{-1}x\right] + k$

Q20. $\frac{x^2}{2}(\tan^{-1}x)^2 - x\tan^{-1}x + \frac{1}{2}\log(1 + x^2) + \frac{1}{2}(\tan^{-1}x)^2 + C$

Q21. $\frac{1}{6}\log\left|\frac{x^2 + 1}{x^2}\right| - \frac{\tan^{-1}x}{3x^3} - \frac{1}{6x^2} + k$

Q22. $\log x\left[\log\log x - 1\right] + k$

Q23. $x(\log x - 1)\log_{10}e + k$

Q24. $\frac{1 + x^2}{2}\log\left|1 + x^2\right| - \frac{x^2}{2} + k$

Q25. $\left(x + \frac{x^3}{3}\right)\log x - \left(x + \frac{x^3}{9}\right) + k$

Q26. $\frac{x}{2} - \frac{x^2}{4} + \frac{x^2 - 1}{2}\log\left|x + 1\right| + k$

Q27. $\left(x + \frac{x^2}{2}\right)\log x - \left(x + \frac{x^2}{4}\right) + k$

Q28. $x\log\left|1 + x^2\right| - 2x + 2\tan^{-1}x + k$

Q29. $x(\log x)^2 - 2x\log x + 2x + k$

Q30. $x\log\left|x + \sqrt{x^2 + a^2}\right| - \sqrt{x^2 + a^2} + k$

Q31. $x\left[\log\log x - \frac{1}{\log x}\right] + k$

Q32. $e^x\tan^{-1}x + k$

Q33. $\frac{e^x}{x} + k$

Q34. $\frac{e^x}{(x - 1)^2} + k$

Q35. $\frac{e^x}{(x - 2)^2} + k$

Q36. $e^x(\log x)^2 + k$

Q37. $\log\left|\sec x + \tan x\right|\, e^x + k$

Q38. $\left(\frac{x - 1}{x + 1}\right)e^x + k$

Q39. $\frac{e^x}{1 + x} + k$

Q40. $\frac{e^x}{1 - x} + k$

Q41. $e^x\tan x + k$

Q42. $e^x\cot 2x + k$

Q43. $-e^x\cot\frac{x}{2} + k$

Q44. $\frac{e^x}{x + 2} + k$

Q45. $\frac{e^x}{x^2 + 1} + k$

Q46. $\frac{1}{2}e^{2x}\cot 2x + k$

Q47. $-e^{-\frac{x}{2}}\sec\left(\frac{x}{2}\right) + k$

Q48. $\frac{e^{2x}}{4x} + k$

Q49. $xe^{\tan^{-1}x} + k$

Q50. $\frac{x}{\log x} + k$

Q51. $\frac{x}{1 + \log x} + k$

Q52. $x\sin\log x + k$

Q53. $\frac{x}{2}\left[\sin\log x - \cos\log x\right] + k$

Q54. $x\tan\frac{x}{2}+k$ Q55. $-x\cot\frac{x}{2}+k$ Q56. $\frac{\sin 2\theta}{2}\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\log\sec 2\theta+k$

Q57. $\frac{1}{3}(2x-1)^{3/2}(3x+4)+k$ Q58. $\frac{1}{2}\left(-\operatorname{cosec}x\cot x+\log\left|\operatorname{cosec}x-\cot x\right|\right)+k$

EXERCISE 6.6

☑ **Type G**

Q01. $2x\tan^{-1}x-\log\left|1+x^2\right|+k$ Q02. $\frac{1}{2}(x\cos^{-1}x-\sqrt{1-x^2})+k$

Q03. $\frac{4}{\pi}\left[x\sin^{-1}\sqrt{x}-\frac{1}{2}\sin^{-1}\sqrt{x}+\frac{1}{2}\sqrt{x}\sqrt{1-x}\right]-x+k$

Q04. $2x\tan^{-1}3x-\frac{1}{3}\log\left|1+9x^2\right|+k$ Q05. $(x+a)\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{ax}+k$

Q06. $\log\left|\frac{x}{x+1}\right|-\frac{\log x}{(x+1)}+C$ Q07. $\frac{1}{2\sqrt{2}}\log\left|\frac{\sin^2 x+1-\sqrt{2}}{\sin^2 x+1+\sqrt{2}}\right|+k$

Q08. $-\frac{2}{\cos(x/2)+\sin(x/2)}+C$ Q09. $\frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right|-\frac{1}{8}\log\left|\frac{1+\sin x}{1-\sin x}\right|+k$

Q10. $\frac{3}{5}\tan^{5/3}x+\frac{3}{11}\tan^{11/3}x+C$ Q11. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x-1}{\sqrt{2}\tan x}\right)+k$

Q12. $\tan^{-1}(\tan^2 x)+k$ OR $\tan^{-1}[2\sin^2 x-1]+k$ OR $-\tan^{-1}\cos 2x+k$

Q13. $2\sqrt{x}-3x^{1/3}+6x^{1/6}-6\log\left|x^{1/6}+1\right|+k$

Q14. $6\left[\frac{\sqrt{1+x}}{3}-\frac{(1+x)^{1/3}}{2}+(1+x)^{1/6}-\log\left|(1+x)^{1/6}+1\right|\right]+k$

Q15. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)-\frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+k$ Q16. $\frac{1}{8}\log\left|\frac{x^2-4x+8}{x^2+4x+8}\right|+k$

Q17. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-4}{2\sqrt{2}x}\right)+k$ Q18. $\frac{1}{6\sqrt{2}}\tan^{-1}\left(\frac{x^2-9}{3\sqrt{2}x}\right)+\frac{1}{12\sqrt{2}}\log\left|\frac{x^2-3\sqrt{2}x+9}{x^2+3\sqrt{2}x+9}\right|+k$

Q19. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-4}{\sqrt{5}x}\right)+\frac{1}{4\sqrt{11}}\log\left|\frac{x^2-\sqrt{11}x+4}{x^2+\sqrt{11}x+4}\right|+k$

Q20. $\sqrt{2}\sin^{-1}(\sin x-\cos x)+k$ OR $\sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+k$

Q21. $\sqrt{2}\log\left|\sin x+\cos x+\sqrt{\sin 2x}\right|+k$

Q22. $\frac{1}{2\sqrt{2}}\log\left|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right|+\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+k$

Q23. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\sin x-\cos x}{\sqrt{3}-\sin x+\cos x}\right|-\tan^{-1}(\sin x+\cos x)+k$

Q24. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\sin x-\cos x}{\sqrt{3}-\sin x+\cos x}\right|+\tan^{-1}(\sin x+\cos x)+k$

Q25. $\tan^{-1}(\sin x-\cos x)+\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\sin x-\cos x}{\sqrt{3}-\sin x+\cos x}\right|+k$

Q26. $\frac{1}{2\sqrt{2}\sqrt{\sqrt{2}+1}}\log\left|\frac{\sqrt{\sqrt{2}+1}+\sin x-\cos x}{\sqrt{\sqrt{2}+1}-\sin x+\cos x}\right|+\frac{1}{\sqrt{2}\sqrt{\sqrt{2}-1}}\tan^{-1}\left(\frac{\sin x-\cos x}{\sqrt{\sqrt{2}-1}}\right)+C$

Q27. $\frac{1}{3\sqrt{2}}\log\left|\frac{\sqrt{2}+\sin x+\cos x}{\sqrt{2}-\sin x-\cos x}\right|+\frac{2}{3}\tan^{-1}(\sin x+\cos x)+C$

Q28. $\frac{1}{3\sqrt{2}}\log\left|\frac{\sqrt{2}+\sin x-\cos x}{\sqrt{2}-\sin x+\cos x}\right|+\frac{2}{3}\tan^{-1}(\sin x-\cos x)+C$

Q29. $\sin x-\cos x+\frac{1}{\sqrt{2}}\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|-\frac{1}{3\sqrt{2}}\log\left|\frac{\sqrt{2}+\sin x-\cos x}{\sqrt{2}-\sin x+\cos x}\right|$
$-\frac{2}{3}\tan^{-1}(\sin x-\cos x)+C$

Q30. $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+k$

Q31. $-\frac{3}{4}\left(1+\frac{1}{x^3}\right)^{4/3}\left[\frac{1}{3}\log\left(1+\frac{1}{x^3}\right)-\frac{1}{4}\right]+k$

Q32. $\frac{1}{2}\left((\sin x-\cos x)+\frac{1}{\sqrt{2}}\log\left|\frac{\tan(x/2)-1-\sqrt{2}}{\tan(x/2)-1+\sqrt{2}}\right|\right)+k$

Q33. $\frac{1}{2}\left[\sin^{-1}\left(\frac{x}{a}\right)+\log\left|x+\sqrt{a^2-x^2}\right|\right]+k$

Q34. $\sqrt{1-x}\,(\sqrt{x}-2)-\sin^{-1}\sqrt{x}+k$

Q35. $\frac{1}{2}x|x|+k$

Q36. $-\log\left|\cot x+\sqrt{\cot^2 x-1}\right|+\sqrt{2}\log\left|\sqrt{2}\cos x+\sqrt{\cos 2x}\right|+k$

Q37. $\frac{\sin x-x\cos x}{x\sin x+\cos x}+k$

Q38. $\frac{1}{2}\log\left|\tan\frac{x}{2}\right|-\frac{1}{4}\tan^2\frac{x}{2}+k$

Q39. $\frac{2\sqrt{\cos a\tan x+\sin a}}{\cos a}-\frac{2\sqrt{\cos a+\sin a\cot x}}{\sin a}+k$

Q40. $\frac{3}{2}\left[\left(\frac{2x+2}{3}\right)\tan^{-1}\left(\frac{2x+2}{3}\right)+\log_e\left|\frac{3}{\sqrt{4x^2+8x+13}}\right|\right]+k$

Q41. $\frac{\sqrt{(b^2-a^2)+2a(a+b\cos x)-(a+b\cos x)^2}}{(b^2-a^2)(a+b\cos x)}$
$-\frac{a}{(b^2-a^2)^{3/2}}\log\left|\left(\frac{1}{(a+b\cos x)}+\frac{a}{b^2-a^2}\right)+\sqrt{\left(\frac{1}{(a+b\cos x)}+\frac{a}{b^2-a^2}\right)^2-\left(\frac{b}{b^2-a^2}\right)^2}\right|+k$

Q42. $-2\log\left|\sqrt{2}\cos\frac{x}{2}+\sqrt{\cos x}\right|+k$

Q43. $\log\left|\sin x+\frac{1}{2}\sqrt{\sin^2 x+\sin x}\right|+k$

Q44. $-\cos\alpha\sin^{-1}\left(\frac{\cos x}{\cos\alpha}\right)-\sin\alpha\,\log\left|\sin x+\sqrt{\sin^2 x-\sin^2\alpha}\right|+k$

Q45. $\log\left|x+\sqrt{x^2-1}\right|-\sec^{-1}x+k$

Q46. $x\log_e\left(\sqrt{1-x}+\sqrt{1+x}\right)+\frac{1}{2}\left(\sin^{-1}x-x\right)+k$

Q47. $2\log|x\sin x+\cos x|-\dfrac{x^2}{x\tan x+1}+k$ Q48. $3\left\{(2-x^{2/3})\cos\sqrt[3]{x}+2\sqrt[3]{x}\sin\sqrt[3]{x}\right\}+k$

Q49. $-\dfrac{1}{2}\log\left|1+\tan^{2/3}x\right|+\dfrac{1}{4}\log\left|\tan^{4/3}x-\tan^{2/3}x+1\right|+\dfrac{\sqrt{3}}{2}\tan^{-1}\left(\dfrac{2\tan^{2/3}x-1}{\sqrt{3}}\right)+k$

Q50. $\dfrac{2}{9}\sin^3\sqrt{x}+\dfrac{2}{3}\sqrt{x}\cos^3\sqrt{x}-2\sqrt{x}\cos\sqrt{x}+\dfrac{4}{3}\sin\sqrt{x}+C$

Q51. $\dfrac{1}{6}\sqrt{x}\sin 3\sqrt{x}+\dfrac{3}{2}\sqrt{x}\sin\sqrt{x}+\dfrac{1}{18}\cos 3\sqrt{x}+\dfrac{3}{2}\cos\sqrt{x}+C$

Q52. $\dfrac{x}{2}+\dfrac{\cos 2x}{8}+\dfrac{\sin 2x}{8}+\dfrac{1}{4}\log|\cos x+\sin x|+C$ Q53. $\dfrac{x}{2}+\dfrac{1}{4\sqrt{2}}\log\left|\dfrac{\sec^2 x+\sqrt{2}\tan x}{\sec^2 x-\sqrt{2}\tan x}\right|+C$

Q54. $\dfrac{1}{2}\left(\dfrac{1}{\sqrt{1+i}}\tan^{-1}\left(\dfrac{\tan x}{\sqrt{1+i}}\right)+\dfrac{1}{\sqrt{1-i}}\tan^{-1}\left(\dfrac{\tan x}{\sqrt{1-i}}\right)\right)+C$

Q55. $\log\left|\dfrac{\sqrt{x^2+x+1}-\sqrt{x}}{\sqrt{x^2+x+1}+\sqrt{x}}\right|+C$ Q56. $\sqrt{x^2+1+\dfrac{1}{x^2}}+C$

Q57. $2\tan^{-1}\left(x+1+\dfrac{1}{x}\right)+C$ Q58. $x+2\sqrt{1+x}-2\log|x+2|-2\tan^{-1}\sqrt{1+x}+C$

Q59. $\dfrac{x}{1+(\log x)^2}+C$.

☑ Type H

Q01. $4x+C$ Q02. $R=\left(\dfrac{x-2}{x+2}\right)e^x+k$ Q03. 1900

Q04. Rs.2000/- Q05. $C(x)=\dfrac{4}{15}(x+5)^{3/2}(3x-10)+300$.

CHAPTER 07

EXERCISE 7.1

☑ Category A

Q01. 4π Q02. $-\dfrac{1}{2}\log 3$ Q03. $\dfrac{2}{3}$ Q04. 1 Q05. $\sqrt{2}-1$

Q06. $\dfrac{1}{2}\log\dfrac{e}{2}$ Q07. $\dfrac{1}{20}\log 3$ Q08. $\dfrac{\pi}{2}$ Q09. $5\log 2-3\log 3$

Q10. $1+\dfrac{4-2\sqrt{2}}{\pi}$ Q11. $\dfrac{e^4}{4}-\dfrac{e^2}{2}$ Q12. $\dfrac{\pi}{8}$ Q13. $\dfrac{\pi}{4}$ Q14. $\dfrac{2}{3}\tan^{-1}\left(\dfrac{1}{3}\right)$

Q15. $\log 2$ Q16. $-\dfrac{3\sqrt{2}}{5}(e^{2\pi}+1)$ Q17. 6 Q18. $\dfrac{1}{8}$ Q19. $\dfrac{2}{3}$

Q20. 2 Q21. –2 Q22. $1-\log 2$ Q23. $\dfrac{64}{231}$ Q24. π

Q25. $\dfrac{a\pi}{2}$ Q26. $\dfrac{6-\sqrt{3}\pi}{12}$ Q27. $\dfrac{\pi}{4}-\dfrac{1}{2}\log 2$ Q28. $\dfrac{\pi}{4}-\dfrac{1}{2}\log 2$ Q29. $\dfrac{\pi}{2}-1$

Q30. $\dfrac{\pi}{8}$ Q31. $\dfrac{(2\sqrt{2}-1)\pi^{3/2}}{12}$ Q32. $2\left(\dfrac{\pi}{\sqrt{3}}-\log 2\right)$ Q33. $\dfrac{\pi}{6}$ Q34. $\dfrac{\pi}{4\sqrt{5}}$

Q35. $\log\left(\frac{4}{3}\right)$ Q36. $\frac{2}{3}$ Q37. $\frac{8}{15}$ Q38. $\frac{3\pi}{16}$ Q39. $\frac{\log 3}{2}$

Q40. $\frac{\pi}{2}-1$ Q41. $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$ Q42. $2\log 3$ Q43. $\frac{\pi}{3}$ Q44. $\frac{1}{2}\log\frac{3}{2}$

Q45. $-\frac{\pi}{2}$ Q46. $\frac{4\sqrt{2}}{3}$ Q47. $\frac{2\pi}{3}$ Q48. $e^{\pi/2}$ Q49. e^e

Q50. $e\left(\frac{e}{2}-1\right)$ Q51. $\frac{e^2}{2}-e$ Q52. $\frac{\log 2}{1+\log 2}$ Q53. $\frac{\pi}{2}$ Q54. $\frac{\pi}{60}$

Q55. $\frac{\pi}{4}-1$ Q56. $\frac{\pi^2-4\pi}{16}+\frac{1}{2}\log 2$ Q57. $\frac{6}{5}$ Q58. $\sqrt{2}\,\pi$

Q59. $\frac{1}{3}\log\left|\sqrt{2}+1\right|+\frac{\pi}{6\sqrt{2}}$

EXERCISE 7.2

☑ Category B

Q01. $2-\sqrt{2}$ Q02. $\frac{13}{2}$ Q03. $\frac{13}{10}$ Q04. $\frac{11}{4}$ Q05. 4

Q06. 2 Q07. 1 Q08. $\frac{3\pi+1}{\pi^2}$ Q09. $2\sqrt{2}$

Q10. $2(\sqrt{2}-1)$ Q11. $\frac{5\pi-2}{2\pi^2}$ Q12. $2\left(1-\frac{1}{e}\right)$ Q13. $\frac{5}{2}$ Q14. 1

Q15. $3(\pi-2)$ Q16. $2-\sqrt{2}$ Q17. 2 Q18. 20 Q19. 17

Q20. 26 Q21. $\frac{e^4+5-\pi}{2}$ Q22. $8+\frac{e^4(e^4-1)}{2}$

EXERCISE 7.3

☑ Category C

Q01. $\frac{\pi}{4}$ Q02. $\frac{\pi}{12}$ Q03. 0 Q04. $\frac{\pi}{4}$ Q05. $\frac{\pi}{2}-\log 2$

Q06. $\log 2$ Q07. 0 Q08. $\frac{\pi}{8}(\log 2)$ Q09. 0 Q10. $\frac{\log(1+\sqrt{2})}{\sqrt{2}}$

Q11. $\frac{\pi}{2\sqrt{2}}\log(1+\sqrt{2})$ Q12. $-\frac{\pi}{2}(\log 2)$ Q13. $\frac{\pi^2}{8}$ Q14. $\frac{\pi}{2}$

Q15. $\frac{\pi}{3\sqrt{3}}$ Q16. 0 Q17. 0 Q18. $\frac{5}{2}$ Q19. $\frac{3}{2}$

Q20. $\frac{1}{(n+1)(n+2)}$ Q21. $\frac{\sqrt{3}\,\pi^2}{9}$ Q22. 0 Q23. 0 Q24. 0

Q25. $\frac{\pi}{4}$ Q26. $\frac{\pi}{8}(\log 2)$ Q27. $\frac{1}{4}$ Q29. $\log 2$ Q30. $\frac{\pi}{12}$

Q31. $\frac{\pi}{2}$ Q34. 2π Q35. (a) 1 Q35. (b) $\frac{8}{3}$

EXERCISE 7.4

☑ Category D

Q01. $a\pi$ Q02. 0 Q03. 0 Q04. 0 Q05. 5π

Q06. 0 Q07. 2 Q08. $2-\sqrt{2}$ Q09. $\log 2$ Q10. 4

Q11. $\dfrac{\sqrt{2}(\pi+4-4\sqrt{2})}{\pi^2}$ Q12. 0 Q13. 0 Q14. $2(e-1)$

Q15. 0 Q16. $2\pi+\dfrac{1}{2a}\sin 2a\pi-\dfrac{1}{2b}\sin 2b\pi$. Also if $a,b\in Z$ then, $I=2\pi$.

Q17. 2 Q18. 1 Q19. 0

EXERCISE 7.5

☑ Category E

Q01. π Q02. $\left(\dfrac{\pi}{2}\right)^2$ Q03. $\dfrac{\pi}{2}(\log 2)$ Q04. $-\dfrac{\pi}{2}(\log 2)$ Q05. $\pi\log\dfrac{1}{2}$

Q06. π^2 Q07. 0 Q08. $\pi\left(\dfrac{\pi}{2}-1\right)$ Q09. $\left(\dfrac{\pi}{2}\right)^2$

Q10. $\dfrac{\pi^2}{2}\log\dfrac{1}{2}$ Q11. $\pi\log 2$ Q12. $\dfrac{\pi}{2}(\log 2)$ Q14. $\dfrac{\pi}{\cos\alpha}\left(\dfrac{\pi}{2}-\alpha\right)$

Q15. $\dfrac{\pi^2}{2ab}$

EXERCISE 7.6

☑ Category F

Q06. 0 Q07. $\dfrac{n}{n^2-1}$ Q08. $-\dfrac{\pi}{2}$ Q09. $\dfrac{\pi}{2}$ Q10. 1

Q11. $\dfrac{(\log_e x)^2}{2}$ Q12. $\dfrac{\pi}{\sqrt{3}}$ Q13. $\dfrac{\pi^2}{6\sqrt{3}}$ Q14. $\dfrac{1}{6}$ Q15. $\dfrac{5\pi}{32}-\dfrac{1}{24}$

Q16. $40\sqrt{2}$ Q17. $\log 2$ Q18. 1.

CHAPTER 08

EXERCISE 8.1

Note that all answers for area **must** be declared in **Square units**.

Q01. $\left(9\pi-4\sqrt{5}-18\sin^{-1}\dfrac{2}{3}\right)$ Sq.units Q02. 9 Sq. units Q03. 6 Sq.units

Q04. $\dfrac{29}{2}$ Sq.units Q05. $\dfrac{3\pi}{2}$ Sq.units Q06. $\dfrac{8a^2}{3}$ Sq. units Q07. $\dfrac{25\pi}{2}$ Sq.units

Q08. $\left(eb^2+ab\sin^{-1}e\right)$ Sq.units Q09. $\dfrac{32}{3}$ Sq.units Q10. $\dfrac{8}{3}$ Sq.units

Q11. $(1,\pm1)$, $\left(\dfrac{10}{3}\cos^{-1}\dfrac{1}{\sqrt{10}}-1\right)$ Sq. units Q12. $\dfrac{5}{2}\left\{\dfrac{2\pi}{3}-\dfrac{\sqrt{3}}{2}\right\}$ Sq.units Q13. 4π Sq. units

Q14. 4 Sq.units Q15. $\dfrac{4}{3}\sqrt{2}$ Sq.units Q16. $\dfrac{5}{24}$ Sq.units Q17. $\left(6+\dfrac{4\pi}{\sqrt{3}}\right)$ Sq.units

Q18. $\dfrac{2}{3}\left[\dfrac{9\pi}{4}-\dfrac{9}{2}\sin^{-1}\dfrac{2}{3}-\sqrt{5}\right]$ Sq.units Q19. $a=4^{2/3}$ Q20. $\dfrac{ab\pi}{4}$ Sq.units

Q21. 4 Sq. units Q22. π Sq.units Q23. 3 Sq. units Q24. $\dfrac{4\sqrt{2}}{3}$ Sq. units

Q25. $\frac{\pi}{3}$ Q26. $\frac{125}{3}$ Sq.units Q27. 16 Sq.units Q28. $\left(\frac{\pi}{4}-\frac{1}{2}\right)a^2$

Q29. $\frac{2}{3}$ Sq.units Q30. 25 Q31. $(\sqrt{2}+1):(\sqrt{2}-1)$ Q32. $\frac{41}{6}$ Sq. units

Q33. $\left(\frac{9\pi}{2}-2\sqrt{5}-9\sin^{-1}\frac{2}{3}\right)$ Sq.units Q34. $\left(\frac{15\pi}{2}-\frac{36}{5}-15\sin^{-1}\frac{3}{5}\right)$ Sq.units

Q35. 12 Sq. units Q36. 1 Sq. units Q37. $-\frac{7}{9}$ Q38. $\frac{4}{3}$ Sq.units

Q39. $\frac{4}{3}$ Sq.units Q40. $\frac{\pi}{2}$ Sq.units Q41. $\frac{11}{24}$ Sq.units Q42. $\frac{2}{3}$ Sq. units

Q43. $\frac{32}{3}$ Sq. units Q44. $\frac{2\pi}{3}$ Sq. units Q45. 12π Sq. units Q46. (a) 26 (b) 20 (c) $\frac{19}{2}$

Q47. $\frac{17}{2}$ Q48. 16 Q49. 9 Q50. 4 Q51. $\frac{13}{6}$ Q52. 2 Q53. $\frac{1}{2}$

Q54. $\frac{4}{3}$ Q55. $\frac{4}{3}$ Q56. $\frac{56}{9}$ Q57. $\frac{14}{9}$ Q58. 4π.

EXERCISE 8.2

Q01. $\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$ Q02. $\frac{4(8\pi-\sqrt{3})}{3}$ Q03. $\frac{23}{6}$ Q04. $6\pi-\frac{9\sqrt{3}}{2}$

Q05. $\sqrt{2}-1$ Q06. $\frac{50}{3}$ Q07. $\frac{4\pi}{3}-\sqrt{3}$ Q08. $\left(\frac{1}{3\sqrt{2}}+\frac{9}{4}\cos^{-1}\frac{1}{3}\right)$ Sq.units

Q09. **(a)** $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ Q09. **(b)** $(\pi-2)$ Q09. **(c)** $\left(\frac{\pi}{4}-\frac{2}{3}\right)a^2$

Q09. **(d)** $\frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}}\right]-\frac{1}{2}$ Q09. **(e)** $\frac{1}{3}$ Q10. $\frac{ab}{2}\left(\frac{\pi}{2}-1\right)$

Q11. 1:3 Q12. $\left(\frac{4\sqrt{3}}{3}+\frac{16\pi}{3}\right)$ Q13. $\frac{4}{3}[8\pi-\sqrt{3}]$ Q14. 4 Q15. 4

Q16. $\frac{4}{3}$ Q17. $\frac{1}{3}$ Q18. $m=2$ Q19. 4 Q20. 4

Q21. $\left(\frac{\sqrt{3}-1}{4}\right)$ Sq. units

Q22. $\frac{3-\sqrt{3}}{2}$ Sq. units.

CHAPTER 09

EXERCISE 9.1

Q01. a) Order: 2, Degree: 1
b) Order: 3, Degree: Not Defined
c) Order: 2, Degree: 1
d) Order: 2, Degree: Not Defined
e) Order: 2, Degree: Not Defined
f) Order: 2, Degree: Not Defined
g) Order: 4, Degree: Not Defined
h) Order: 2, Degree: 1
i) Order: 3, Degree: 1
j) Order: 3, Degree: Not Defined
k) Order: 2, Degree: Not Defined.

Q07. (a) 2 Q07. (b) 4 Q07. (c) 3.

EXERCISE 9.2

Q01. $y = e^{3x} + k$

Q02. $x = \frac{y^3}{3} - \cos y + k$

Q03. $y = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan x - \cot x}{\sqrt{2}}\right) + k$

Q04. $y + x = 2\tan\frac{x}{2} + k$

Q05. $\tan\frac{y}{2} = \cot\frac{x}{2} + k$

Q06. $y = \left(\frac{x^2+1}{2}\right)\tan^{-1} x - \frac{x}{2} + k$

Q07. $y\sqrt{1-x^2} + x\sqrt{1-y^2} = k$

Q08. $2y = x + \log|x - y + 2| + k$

Q09. $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+5}{2}\right) = x + k$

Q10. $\log\left|1 + \tan\frac{x+y}{2}\right| = x + k$

Q11. $x + \tan\left(\frac{\pi}{4} - \frac{x+y}{2}\right) = k$

Q12. $x = \tan(x - 2y) + k$

Q13. $\log|xy| = 2y + \frac{1}{x} - \frac{y^2}{2} + k$

Q14. $y = \frac{1}{6}(x^6+1)\tan^{-1}(x^3) - \frac{x^3}{6} + k$

Q15. $\log|1+y| = x + \frac{x^2}{2} + k$

Q16. $\log|1+y| = x - \frac{x^2}{2} + k$

Q17. $y - x = \log|x(1+y)| + k$

Q18. $e^y = e^x + \frac{x^3}{3} + k$

Q19. $3(e^x + e^{-y}) + x^3 = k$

Q20. $y = -x + (x+1)\log|x+1| + k$

Q21. $\frac{e^{ax}}{a} + \frac{e^{-by}}{b} + k = 0$

Q22. $e^x(x-1) = \sqrt{1-y^2} + k$

Q23. $\sin y = e^x \log x + k$

Q24. $y = x^2$

Q25. $ye^x = 1$

Q26. $|\tan x \tan y| = k$

Q27. $y = \tan\left[\frac{\pi}{4} + \frac{1}{2} - \frac{[1+\log|x|]^2}{2}\right]$

Q28. $y = \tan^{-1}[C(1-e^x)^3]$

Q29. $\tan^{-1}\left(\frac{x+y}{1-xy}\right) + \frac{1}{2}\log\{(1+x^2)(1+y^2)\} = k$

Q30. $(1+\sin x)(1+\cos y) = k$

Q31. $\frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + \sqrt{1+x^2} + \sqrt{1+y^2} = k$

Q32. $(x+a)(1-ay) = ky$

Q33. $y = \frac{1}{2}\log\left|\frac{x-y-1}{x-y+1}\right| + k$

Q34. $y + x = \frac{1}{y} + \frac{1}{x} + k$

Q35. $(x-y) + \log|xy| = k$

Q36. $e\,x\,y = e^x$

Q37. $y = e^{\sin^2 x}$

Q38. $y = \log|x + y + 1|$

Q39. $x - y = e^{x+y+1}$

Q40. $(1+\log x)^2 - 2\log|1-y^2| = 1$

EXERCISE 9.3

Q01. $y^2 + 2xy - x^2 = C^2$

Q02. $\tan\left(\frac{y}{x}\right) + \log|x| = k$

Q03. $y - 2x = k\,x^2 y$

Q04. $\cos\left(\frac{y}{x}\right) + \log|x| + k = 0$

Q05. $y = xe^{kx}$

Q06. $\log|y| - \frac{x^3}{3y^3} = k$

Q07. $y+\sqrt{x^2+y^2}=kx^2$ Q08. $2e^{x/y}=k-\log y$ Q09. $\sin(y/x)=kx$

Q10. $\frac{y}{2x}-\frac{1}{4}\sin\left(\frac{2y}{x}\right)+\log|x|=k$ Q11. $ky=e^{y/x}$ Q12. $x^2+y^2=kx$

Q13. $4\log|x|=\frac{y}{x}-\log\left|\frac{y}{x}\right|+k$ Q14. $(y^2+x^2)^2=\lambda(y^2-x^2)$ Q15. $y+\sqrt{y^2-x^2}=C$

Q16. $ky=\log\left|\frac{y}{x}\right|-1, x\neq 0$ Q17. $x+ye^{x/y}=k$ Q18. $y=\frac{x\log|x|}{1-\log|x|}$

Q19. $x^2+y^2=2x$ Q20. $(2y^2+x^2)x^2=3$ Q21. $y=\frac{2x}{1+\log|x|}$

Q22. $e^{-y/x}\left[\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right]=1+2\log x, x\neq 0$ Q23. $y=x\cos^{-1}[\log|x|]$

Q24. $3|x^2y|=|y+2x|$ OR $3x^2y=\pm(y+2x)$ Q25. $x^4+6x^2y^2+y^4=8$

Q26. $\cot\left(\frac{y}{x}\right)=\log|ex|$

Q27. $e^{x/y}=y+k$ #**Note** that this differential equation is solved using the concept of homogeneous differential equation, but it isn't a homogeneous differential equation.

Q28. $2-\log y=2e^{\frac{x}{y}}$ Q29. $\log|x|=\cos\left(\frac{y}{x}\right)$ Q30. $\log x=e^{-1}-e^{-y/x}$

Q31. $x^2=2y^2\log|y|$ Q32. $\left(x+ye^{x/y}\right)y^2=ky$

EXERCISE 9.4

Q01. $y=\frac{1}{2}e^{\sin x}+ke^{-\sin x}$ Q02. $y=e^x\left(\frac{x^2}{2}+k\right)$ Q03. $xy=\frac{y^4}{4}+k$

Q04. $y=2(x\tan^{-1}x+1)+k\sqrt{1+x^2}$ Q05. $ye^{\tan x}=(\tan x-1)e^{\tan x}+k$

Q06. $xe^{\tan^{-1}y}=e^{\tan^{-1}y}(\tan^{-1}y-1)+k$ Q07. $x=y^3+ky$ Q08. $y=x^3+kx$

Q09. $ye^{2x}=-\frac{5}{4}e^{-x}+k$ Q10. $x=\sin^{-1}y-1+ke^{-\sin^{-1}y}$

Q11. $y=\tan^{-1}x-1+ke^{-\tan^{-1}x}$ Q12. $xe^{\tan^{-1}y}=\tan^{-1}y+k$

Q13. $y(\sec x+\tan x)=\sec x+\tan x-x+k$ Q14. $y=x(k+x^2)$

Q15. $y=\sin x+\frac{k}{x}$ Q16. $y=\frac{2k-x^2}{2(1+\sin x)}$ Q17. $y(\log x)=\frac{1}{2}(\log x)^2+k$

Q18. $ye^{\tan^{-1}x}=\frac{1}{2}e^{2\tan^{-1}x}+k$ Q19. $(x^2-1)y=\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+k$ Q20. $x=y(k+3y)$

Q21. $\frac{y}{x^2+1}=x+\tan^{-1}x+k$ Q22. $y=x\,ln\,x+x$ Q23. $y\log x+\frac{2}{x}(1+\log x)=k$

Q24. $y=(2\sqrt{x}+k)e^{-2\sqrt{x}}$ Q25. $xy+\tan^{-1}x=k$ Q26. $e^{x^2}\tan y=\frac{x^2-1}{2}e^{x^2}+k$

Q27. $(x+1)y=2(e^x-1)$ Q28. $\frac{x}{y^2}+e^{-y}=e^{-1}$ Q29. $4xy-2=x^2(2\log x-1)$

Q30. $y^2=\sin x$ Q31. $3y(1+x^2)=4x^3$ Q32. $2y=\sin x$

Q33. $xe^{\tan^{-1}y}=\frac{1}{2}e^{2\tan^{-1}y}+C$ Q34. $y=(x+1)\left(\frac{e^{3x}}{9}[3x+2]+C\right)$

Q35. $x(\cos y+\sin y)=\sin y+\frac{C}{e^{y}}$ Q36. $xy\sin x=\sin x-x\cos x+C$

Q38. $y=x^{2}+xC$ Q39. $y=x\log|x|+xC$

EXERCISE 9.5

Q01. $y=\frac{1}{1+2x^{2}}$ Q03. $y=x^{2}-\frac{\pi^{2}}{4\sin x}$; $x\neq n\pi,\ n\in Z$

Q04. $\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)-\frac{1}{2}\log\left|y^{2}+xy+x^{2}\right|=C$ Q05. $\sin\left(\frac{y}{x}\right)=\log|x|+C$

Q06. $(x-y)^{2}=kxe^{-y/x}$ Q07. $\sec\left(\frac{y}{x}\right)=kxy$ Q08. $\left|xy\cos\left(\frac{y}{x}\right)\right|=k$

Q09. $(2-e^{y})(x+1)=1$ Q10. $27y=27x^{3}-2\pi^{3}\cos x$ Q11. $\frac{2}{3}$ Q12. 7

Q13. $4e^{3x}+3e^{-4y}=7$ Q14. $x_{0}=\pm\sqrt{3}e$ Q15. $2y=\sin x-\cos x+ce^{x}$

Q16. $(\log y+1)+yx^{2}\cos x+Cy=2y(x\sin x+\cos x)$ Q17. $2y\sin y=2x^{2}\log x+\pi$

Q18. $y=\tan x-\sqrt{\tan x}$ Q19. (a) $1+y^{2}$ Q19. (b) $e^{2\sqrt{x}}$ Q19. (c) $e^{\tan^{-1}y}$

Q20. $xy=\frac{\pi}{4}-\tan^{-1}x$ Q21. $ye^{\tan^{-1}x}=\frac{1}{(m+1)}e^{(m+1)\tan^{-1}x}+\frac{m}{m+1}$ Q22. $y^{2}=2x^{2}\sin\frac{y}{x}+\lambda$

Q23. $\frac{1}{y^{2}}=2\left(1+\frac{x^{2}}{2}\right)-ce^{\frac{x^{2}}{2}}$ Q24. $\sqrt{x^{2}-y^{2}}+x=C$ Q25. $y=f(x)-1+C\times e^{-f(x)}$

Q26. $\cos x=y(C-x)$

EXERCISE 9.6

Q01. $y=\sqrt[3]{3x^{2}+15}$ Q03. $2\log_{e}(10)$ units Q04. $xy=100$

Q05. $v=V\log\left(\frac{m_{0}}{m}\right)$ Q06. $x=2y^{2}$, 8.

CHAPTER 10

EXERCISE 10.1

Q01. Maximum value of z is 19 at $\left(\frac{7}{2},\frac{3}{4}\right)$; minimum value of z is 3 at $\left(0,\frac{3}{2}\right)$.

Q02. Minimum value of z is 26 Q03. Maximum value of z is $22\frac{8}{13}$ at $\left(\frac{30}{13},\frac{6}{13}\right)$.

Q04. Maximum value of z is 10 Q05. Max. Value = 16

Q06. Max. Value = 16 at (2, 4) Q07. Min. Value = 12 at (4, 0)

Q08. Max. $Z=400$ at $(0,200)$

Q09. (i) Max. $Z=12$ at $E(4,0)$, Min. $Z=-32$ at $A(0,8)$

(ii) $p=q$; number of optimal solutions is infinite.

Q10. (i) Maximum value of Z is 43 at B(3, 4)

(ii) Value of $p=\frac{2}{7}$; change in the value of $p=\frac{2}{7}$.

Q11. Minimum value of Z is 60 at B(5, 5).

Maximum value of Z is 180 and it occurs at C(15, 15) and D(0, 20). Note that, this value of $Z=180$ occurs at each point of the line segment joining the points C and D.

Q12. Maximum value of Z is 17.4, when $x=10.2,\ y=7.2$.
But number of books can not be in decimals.
$\therefore$ The shelf can carry a maximum of 10 and 7 books of type I and type II respectively.

Q13. Maximum value of Z is 2412.

Q14. **(i)** Maximum value of z is $22\frac{8}{13}$. **(ii)** Maximum value of z is attained at $\left(\frac{30}{13},\frac{6}{13}\right)$.
(iii) Possible constraints for the given feasible region are
$x\ge 0,\ y\ge 0,\ y\le 1,\ 2x+3y\le 6,\ 3x-2y\le 6$.

Q15. Maximum value of Z is 69 at $x=2,\ y=3$; maximum value of Z is 5.

Q16. Minimum value of Z is –35.

Q17. **(i)** The minimum value of F occurs at any point on the line segment joining the points (0, 2) and (3, 0). Also, the maximum value of F occurs at (6, 8).
Required difference between Maximum and Minimum value of $F=60$.
(ii) $3m-5n=0$.

Q18. Max. value of Z is 230 at (2, 3). Q19. Minimum value of Z is 40.

Q20. **(i)** $x\ge 0,\ \ y\ge 0,\ 5x+8y\le 200,\ 10x+8y\le 240$.
(ii) Maximum value of Z is 1600 and it occurs at the corner point $B(8, 20)$.
Minimum value of Z is 1200 and it occurs at the corner point $C(24, 0)$.

Q21. Minimum value of Z is -6 at (0, 6).

Q22. Maximum value of $Z=6750$, when $x=20$ and $y=15$.

Q23. Maximum value of Z is 168000, when $x=12$ and $y=6$.

Q24. (i) Maximum value of Z is 14. (ii) (7, 0).
(iii) $x\ge 0,\ y\ge 0,\ x+y\le 7,\ 2x-3y+6\ge 0,\ y\le 3$.

Q25. (i) Minimum value of Z is 5. (ii) Maximum value of Z is 32 at (3, 4).
(iii) $14m-5n=0$.

Q26. Minimum value of Z is 6; maximum value of Z is 63.

EXERCISE 10.2

Q01. Let the number of necklace and bracelets made per day be x and y respectively.
To maximize: $Z=₹(100x+300y)$
Subject to constraints : $x+y\le 24,\ x+2y\le 32,\ x\ge 1,\ y\ge 1$.

Q02. Let the distance covered with speed of 25 km/h be x km and the distance covered with speed of 40 km/h be y km.
To maximize : $Z=x+y$
Subject to constraints :
$x\ge 0,\ y\ge 0,\ 4x+5y\le 200,\ 8x+5y\le 200$.

Q03. Let x be the number of cakes of one kind and y be the number of cakes of other kind.
To maximize: $Z=(x+y)$
Subject to constraints: $2x+y\le 50,\ x+2y\le 40,\ x\ge 0,\ y\ge 0$.

Q04. Maximum profit : ₹510; 16 small cylinders and 3 large cylinders.

CHAPTER 11

EXERCISE 11.1

Q02. $-4(\hat{i}+\hat{j}+\hat{k})$	Q03. $-7,6,0;\ -7\hat{i},6\hat{j},0\hat{k}$	Q04. $\sqrt{38}$	Q05. 0, 1
Q06. $x=2,y=2,z=1$	Q11. $\alpha=3,\ \beta=-1$	Q13. 2	Q14. ± 12

Q16. $\frac{2}{3}$

Q17. $\frac{1}{\sqrt{14}}\hat{i}+\frac{2}{\sqrt{14}}\hat{j}+\frac{3}{\sqrt{14}}\hat{k}$

Q18. $\frac{-3}{\sqrt{29}}\hat{i}+\frac{4}{\sqrt{29}}\hat{j}+\frac{2}{\sqrt{29}}\hat{k}$

Q19. $\frac{16}{\sqrt{5}}\hat{i}-\frac{8}{\sqrt{5}}\hat{j}$

Q20. $\frac{4}{\sqrt{29}}\hat{i}+\frac{3}{\sqrt{29}}\hat{j}-\frac{2}{\sqrt{29}}\hat{k}$

Q21. $\frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k}),-\frac{1}{\sqrt{69}}(\hat{i}+2\hat{j}-8\hat{k})$

Q22. $\pm\frac{1}{\sqrt{3}}$

Q23. $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$

Q24. $-\frac{1}{3},-\frac{2}{3},\frac{2}{3};\ -2,-4,4$

Q28. $\theta=\frac{\pi}{3},\ \hat{a}=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}$

Q30. (a) $\cos\theta\,\hat{i}+\sin\theta\,\hat{j}$ (b) $\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}$

Q31. $\frac{3\sqrt{3}\hat{j}-5\hat{i}}{2}$

Q32. Two vectors

Q33. $\frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}}$

Q34. $\frac{1}{2}$

Q35. $\frac{5\hat{i}}{2}+\frac{5\sqrt{3}\,\hat{j}}{2}$

Q36. $1, 1, -2;\ \frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},-\frac{2}{\sqrt{6}}$.

EXERCISE 11.2

Q01. $-\vec{a}-7\vec{b}$

Q02. **(i)** $\frac{7\vec{a}+4\vec{b}}{3}$ **(ii)** $\vec{a}+8\vec{b}$

Q03. $\frac{\vec{a}+2\vec{b}}{3}$

Q04. **(i)** $\frac{5\vec{a}}{3}$ **(ii)** $4\vec{b}-\vec{a}$

Q05. $3\hat{i}+2\hat{j}+\hat{k}$

Q09. $3\vec{a}+5\vec{b}$

Q11. $2:3$

Q12. $5\hat{i}+\frac{14}{3}\hat{j}-6\hat{k}$

Q13. $\frac{5\vec{a}+3\vec{b}}{8}$

Q14. $\overrightarrow{OC}=2\vec{b}-\vec{a}$

Q15. $\frac{\sqrt{34}}{2}$ units

Q16. $3\vec{a}+4\vec{b}$.

EXERCISE 11.3

Q02. 46

Q04. $\cos^{-1}\left(\frac{1}{3}\right)$

Q05. 3

Q06. 0

Q07. 2

Q08. $\lambda=5$

Q09. $\sqrt{5}$

Q10. π

Q11. $\angle A=\cos^{-1}\sqrt{\frac{35}{41}},\ \angle B=\cos^{-1}\sqrt{\frac{6}{41}}$

Q12. $-\frac{31}{12},\frac{41}{12}$

Q13. $\frac{16}{3}\sqrt{\frac{2}{7}},\ \frac{2}{3}\sqrt{\frac{2}{7}}$

Q14. 3

Q15. $-\hat{i}-\hat{j}-\hat{k},\ 7\hat{i}-2\hat{j}-5\hat{k}$

Q16. $\vec{\beta}_1=\frac{3}{2}\hat{i}-\frac{1}{2}\hat{j},\ \vec{\beta}_2=\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}$

Q17. $\vec{b}=(4\hat{i}-2\hat{j}-4\hat{k})+(3\hat{i}+4\hat{j}+\hat{k})$

Q18. $\pm\sqrt{2}$

Q19. 4

Q20. $\frac{5(\hat{i}-2\hat{j}+\hat{k})}{6}$

Q21. $3\hat{i}+2\hat{k}$

Q22. $\hat{i}+2\hat{j}+\hat{k}$

Q23. $\frac{\pi}{6}$

Q24. $\frac{\pi}{3}$

Q27. -25

Q28. $-\frac{3}{2}$

Q29. $|\vec{a}|=|\vec{b}|=1$

Q31. $\sqrt{3}$

Q33. 120° or, $\frac{2\pi}{3}$

Q34. $5\sqrt{2}$

Q35. -10

Q38. $\cos^{-1}\left(\frac{1}{3}\right)$

Q40. -14

Q42. $\cos^{-1}\left(-\frac{1}{3}\right)$

Q43. $\lambda=\frac{16}{5}$

Q44. $\frac{8}{7}$

Q45. $\lambda=1$

Q46. $-\frac{21}{2}$

Q47. 3

Q48. $\alpha\in(0, 1)$

Q49. 40 units

Q50. 6 units.

EXERCISE 11.4

Q01. 180° (or, π)

Q02. 45°

Q03. $2\hat{k}-\hat{i}-\hat{j}$

Q04. $2a^2$

Q05. $6\sqrt{21}$

Q06. $\lambda=3,\ \mu=\frac{27}{2}$

Q07. $\mu=27,\ \lambda=-9$

Q08. $15\sqrt{2}$ Sq.units

Q09. $5\sqrt{3}$ Sq.units

Q10. $4\hat{i}+5\hat{k}, -2\hat{i}-2\hat{j}-3\hat{k}$ (or, $2\hat{i}+2\hat{j}+3\hat{k}$) Q11. $\frac{\sqrt{21}}{2}$

Q12. $\sin^{-1}\left(\frac{1}{\sqrt{51}}\right), \frac{1}{\sqrt{2}}$ Sq.units (if **dot product** is used, then the required angle will be $\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$

Q13. $\frac{11\sqrt{5}}{7}[3\hat{i}-6\hat{j}+2\hat{k}], 11\sqrt{\frac{5}{69}}[2\hat{j}-\hat{i}-8\hat{k}]$ Q14. 2 Sq. units Q15. $\frac{\pi}{4}$

Q16. $\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{i}-\frac{1}{\sqrt{6}}\hat{k}$ Q17. $\frac{3}{4}$ Q18. $\frac{1}{4}\hat{i}+\frac{1}{4}\hat{j}+\frac{3}{4}\hat{k}$ Q19. $7(\hat{i}-\hat{j}-\hat{k})$

Q20. $\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$ Q25. 60 Q26. 4 Q27. 1

Q30. $\frac{-3\hat{i}+5\hat{j}+11\hat{k}}{\sqrt{155}}, \sin\theta=\sqrt{\frac{155}{156}}$

Q31. **(a)** $\frac{1}{\sqrt{2}}(-3\hat{i}+8\hat{j}+5\hat{k})$ **(b)** $\pm\left(\frac{\hat{j}-\hat{i}}{\sqrt{2}}\right)$ **(c)** $\pm\left(\frac{\hat{k}-\hat{j}}{\sqrt{2}}\right)$ **(d)** $\pm\left(\frac{4\hat{k}-10\hat{i}-7\hat{j}}{\sqrt{165}}\right)$

Q32. $p=-8, q=4, r=2, s=-11$ or, $p=8, q=4, r=2, s=5$

Q34. $\hat{d}_1=\frac{2\hat{i}-\hat{j}-\hat{k}}{\sqrt{6}}$ and $\hat{d}_2=\frac{-3\hat{j}-4\hat{k}}{5}$ or, $\frac{3\hat{j}+4\hat{k}}{5}$; $2\sqrt{101}$ Sq.units

Q35. $\frac{\sqrt{210}}{2}$ Sq.units Q36. $\sqrt{42}$ units^2 Q37. $3\hat{i}+2\hat{j}+2\hat{k}$ Q38. 1 Q39. 1

Q40. 4 Sq.units Q41. $-11\hat{i}+122\hat{j}-85\hat{k}$; $-46\hat{i}+66\hat{j}+34\hat{k}$ Q42. $\pm(\hat{i}-11\hat{j}-7\hat{k})$.

EXERCISE 11.5

Q01. [0, 12] Q02. $\sqrt{3}$ Q03. $\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}$ Q05. 4 Q06. 0

Q08. 0 Q11. 3 units Q13. $\frac{5\sqrt{3}}{2}, \frac{5}{2}$ Q14. $\pm\frac{1}{3}$ Q15. $60^\circ=\frac{\pi}{3}$

Q16. (6, 11) Q17. $-\frac{11}{15}\hat{i}-\frac{10}{15}\hat{j}-\frac{2}{15}\hat{k}$ Q19. $\theta\in\left(\frac{2\pi}{3}, \pi\right)$ Q20. $5\hat{i}+5\hat{j}+5\hat{k}$

Q21. $-\frac{4\hat{i}+5\hat{j}-\hat{k}}{8}$ Q22. $\alpha=-\frac{3}{2}, \beta=1, \gamma=-\frac{5}{2}$ Q24. $\sqrt{61}$ Q25. $\cos^{-1}\left(\frac{2}{3}\right)$.

EXERCISE 11.6

Q01. $\vec{a}.(\vec{b}\times\vec{c})=36, (\vec{a}\times\vec{b}).\vec{c}=36$ Q03. (a) 6 (b) 6 Q04. (a) 2

Q07. $\lambda=5$ Q09. $x=-2$ Q10. 0

CHAPTER 12

EXERCISE 12.1

Q01. (3, 2, 0) Q02. –1 Q03. $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Q04. $\mp\frac{9}{11}, \pm\frac{6}{11}, \mp\frac{2}{11}$ Q06. For x-axis : 1,0,0 ; y-axis : 0,1,0 and for z-axis : 0,0,1 .

Q07. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$ Q08. $\sqrt{34}$ units Q09. $\pm\frac{2}{3}, \mp\frac{1}{3}, \mp\frac{2}{3}$.

EXERCISE 12.2

Q01. $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-2\hat{j}+3\hat{k})$; $x=1+\lambda, y=2-2\lambda, z=3+3\lambda$; $\frac{x-1}{1}=\frac{y-2}{-2}=\frac{z-3}{3}$

Q02. $\vec{r}=(-2\hat{i}+\hat{j}+5\hat{k})+\lambda(\hat{i}+\hat{j}-\hat{k})$; $\vec{r}=(\hat{i}+4\hat{j}+2\hat{k})+\lambda(\hat{i}+\hat{j}-\hat{k})$

Q03. $\vec{r}=2\hat{i}-\hat{j}+\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$; $\frac{x-2}{2}=\frac{y+1}{-2}=\frac{z-1}{1}$

Q04. $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z-3}{-2}$, $\vec{r}=2\hat{i}-\hat{j}+3\hat{k}+\lambda(2\hat{i}+3\hat{j}-2\hat{k})$

Q06. $\vec{r}=2\hat{i}-\hat{j}+\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ Q07. $\vec{r}=\vec{0}+\lambda(5\hat{i}-2\hat{j}+3\hat{k})$; $\frac{x}{5}=\frac{y}{-2}=\frac{z}{3}$ Q08. $\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$

Q09. $\left(-\frac{1}{3},\frac{1}{3},1\right)$; 2, 1, –6; $\vec{r}=\left(-\frac{1}{3}\hat{i}+\frac{1}{3}\hat{j}+\hat{k}\right)+\lambda(2\hat{i}+\hat{j}-6\hat{k})$ Q10. $\frac{4}{5},-\frac{3}{5},0$

Q11. 1, 2, 3; $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+1}{3}$; $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$ Q12. $\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$

Q13. $\vec{r}=2\hat{i}-3\hat{j}+4\hat{k}+\lambda(-\hat{i}+13\hat{j}-19\hat{k})$ Q14. $\vec{r}=(2\hat{i}+3\hat{j}+4\hat{k})+\lambda\hat{k}$

Q15. $\vec{r}=2\hat{i}-\hat{j}+3\hat{k}+\mu(2\hat{i}+\hat{j}-2\hat{k})$ Q16. $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(-4\hat{i}+2\hat{j})$

Q17. $\vec{r}=2\hat{i}-\hat{j}+4\hat{k}+\lambda(\hat{i}+2\hat{j}-\hat{k})$ Q18. $\vec{r}=-3\hat{i}+5\hat{j}-6\hat{k}+\lambda(\hat{i}+2\hat{j}+\hat{k})$

Q19. $\vec{r}=3\hat{i}-2\hat{j}-5\hat{k}+\lambda(0\hat{i}+0\hat{j}+11\hat{k})$; $\frac{x-3}{0}=\frac{y+2}{0}=\frac{z+5}{11}$.

EXERCISE 12.3

Q01. $\frac{\pi}{3}$ Q02. (a) $\frac{1}{2},\frac{1}{3},\frac{1}{4}$; $\frac{6}{\sqrt{61}},\frac{4}{\sqrt{61}},\frac{3}{\sqrt{61}}$ Q02. (b) 0, 3, –1 Q03 $-\frac{2}{\sqrt{30}},\frac{1}{\sqrt{30}},-\frac{5}{\sqrt{30}}$

Q07. The d.c's of sides of triangle: $-\frac{2}{\sqrt{17}},-\frac{2}{\sqrt{17}},\frac{3}{\sqrt{17}};\frac{4}{\sqrt{42}},\frac{5}{\sqrt{42}},-\frac{1}{\sqrt{42}};-\frac{2}{\sqrt{17}},-\frac{3}{\sqrt{17}},-\frac{2}{\sqrt{17}}$.

Also, angles of the triangle are: $\cos^{-1}\left(\sqrt{\frac{21}{34}}\right)$; $\cos^{-1}\left(\frac{4}{17}\right)$; $\cos^{-1}\left(\sqrt{\frac{21}{34}}\right)$

Q08. (a) $\cos^{-1}\left(\frac{19}{21}\right)$ (b) $\cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$ (c) $\cos^{-1}\left(\frac{5\sqrt{6}}{18}\right)$ Q09. $\cos^{-1}\left(\frac{2}{3}\right)$

Q10. $\frac{\pi}{2}$ Q12. 0° Q14. $\frac{70}{11}$ Q18. 4

Q19. $\frac{6}{7},\frac{2}{7},-\frac{3}{7}$ or, $-\frac{6}{7},-\frac{2}{7},\frac{3}{7}$ Q20. 90°.

EXERCISE 12.4

Q01. (a) $\frac{10}{\sqrt{59}}$ (b) $\frac{\sqrt{293}}{7}$ (c) $\frac{91}{\sqrt{30}}$ (d) $\frac{8}{\sqrt{29}}$ (e) $2\sqrt{5}$

Q02. $(-2,-1,3)$, $\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$ Q03. $\sqrt{\frac{5}{29}}$ units Q04. $\frac{x}{1}=\frac{y}{0}=\frac{z}{2}$; $\frac{x-1}{-1}=\frac{y-3}{0}=\frac{z}{0}$; 3 units

Q05. Coordinates : $P(5,4,0)$ and $Q(-1,-2,-3)$; S.D. = 9 units. Eq. of S.D. : $\frac{x-5}{2}=\frac{y-4}{2}=\frac{z-0}{1}$.

Q06. $\frac{3}{2}\sqrt{2}$ units; $\frac{6x-17}{1}=\frac{6y-1}{0}=\frac{17-6z}{1}$ Q07. 14 units; $\frac{x-5}{2}=\frac{y-7}{3}=\frac{z-3}{6}$; $(5,7,3),(9,13,15)$

Q08. $\vec{r}=3\hat{i}+8\hat{j}+3\hat{k}+\lambda(6\hat{i}+15\hat{j}-3\hat{k})$ Q09. (a) $(-1,-1,-1)$ Q09. (b) (4, 0, –1) Q10. $\lambda=5$

Q12. $(3a,2a,3a)$ and (a,a,a) Q14. $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$

Q15. $\vec{r}=(2\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-\hat{j}+\hat{k})$; $\sqrt{\frac{11}{6}}$ units Q16. $\frac{x+2}{1}=\frac{y-1}{0}=\frac{z-3}{-1}$ Q17. $\sqrt{\frac{14}{3}}$ units

Q18. $(1,0,7)$ Q19. (–2, 1, 7), (–3, –6, 10) Q20. $\hat{i}+2\hat{j}+3\hat{k}$ Q21. $\sqrt{10}$ units

Q22. $\frac{x-1}{0}=\frac{6-y}{3}=\frac{z-3}{2}$; Length of $\perp^{er}=\sqrt{13}$ units Q23. $(3,4,5)$

Q24. 7; the lines are non-intersecting. Q25. 1; the given lines are not intersecting.

Q26. $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$; $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$ Q27. $a\neq 3$ Q28. $m=5$

Q29. $\vec{r}=(\hat{i}-\hat{j}+\hat{k})+\lambda(10\hat{i}-4\hat{j}-7\hat{k})$ Q31. P(1, –2, 7), Ratio is 2 : 1 externally. Q32. $(3,-1,1)$

Q33. 13 units Q34. 7 units Q35. (2, 3, –1); $\sqrt{21}$ units

Q36. $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$; (–4, 1, –3) Q37. $5\hat{i}+4\hat{j}+4\hat{k}$; $4\sqrt{2}$ units.

EXERCISE 12.5

Q01. $\left(\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)$ Q02. $(4,0,-1)$ Q03. $k=-1$ Q04. $k=\frac{9}{2}$ Q05. $\frac{x-1}{3}=\frac{y-1}{10}=\frac{z-1}{17}$

Q07. $5\hat{i}+4\hat{j}$ and $-\hat{i}-2\hat{j}-3\hat{k}$; 9 units Q08. $\frac{x}{2}=\frac{y}{-4}=\frac{z}{4}$; 60 km, $10\sqrt{3}$ km

Q09. $\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-6}{6}$ Q10. $p=2$; (1, 3, 5) Q11. $\frac{x-1}{-13}=\frac{y-3}{2}=\frac{z-5}{3}$

Q12. $k=-2$; $\vec{r}=3\hat{i}-4\hat{j}+7\hat{k}+\lambda(-26\hat{i}+33\hat{j}+27\hat{k})$ Q13. $\frac{x-1}{1}=\frac{y+2}{-1}=\frac{z-5}{-1}$.

CHAPTER 13

EXERCISE 13.1

Q01. 0.4 Q02. $\frac{2}{5}$ Q03. $\frac{33}{40}$ Q04. $\frac{4}{7}$ Q05. $\frac{1}{3},\frac{1}{2}$ Q06. $\frac{5}{9}$ Q07. 0.1

Q08. (a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ Q09. (i) 0.5 (ii) 0.05 Q10. 0.12, 0.6 Q11. $\frac{1}{2}$

Q12. $\frac{1}{3}$ Q13. $\frac{1}{3}$ Q14. $\frac{1}{3}$ Q15. $\frac{1}{3},\frac{1}{9}$ Q16. $\frac{3}{5}$ Q17. $\frac{1}{4}$

Q18. $\frac{1}{17}$ Q19. $\frac{1}{3}$ Q20. $\frac{2}{9}$ Q21. $\frac{1}{3}$ Q22. $\frac{1}{2}$ Q23. (a) $\frac{3}{5}$ (b) $\frac{1}{2}$.

EXERCISE 13.2

Q01. $\frac{3}{4}$ Q02. (a) 0.4 (b) 0.1 (c) $\frac{13}{30}$ (d) $\frac{1}{60}$ Q03. $\frac{3}{5}$ Q04. $\frac{16}{121},\frac{49}{121},\frac{56}{121}$

Q05. $\frac{2}{663}$ Q06. 42% Q07. $\frac{2}{3},\frac{1}{2}$ Q08. A and B are independent events.

Q09. $P(A)=\frac{1}{5},\frac{5}{6}$ and $P(B)=\frac{1}{6},\frac{4}{5}$ Q12. $P(A)=\frac{1}{2},\frac{1}{4}$ and $P(B)=\frac{1}{4},\frac{1}{2}$ Q13. $\frac{1}{3}$

Q14. (i) $\frac{13}{15}$ (ii) $\frac{49}{90}$ Q15. $\frac{1}{5}$ Q16. (a) $\frac{1}{28}$ (b) $\frac{18}{28}$ (c) $\frac{10}{28}$

EXERCISE 13.3

Q01. $\frac{1}{2}$ Q02. $\frac{1}{8}$ Q03. $\frac{1}{17}$ Q04. $\frac{19}{42}$ Q05. $\frac{1}{2}$ Q06. $\frac{29}{63}$

Q07. $\frac{673}{1260}$ Q08. $\frac{11}{21}$ Q09. 0.27 Q10. 0.0345 Q11. $\frac{19}{42}$ Q12. 0.016

Q13. $\frac{22}{45}$ Q14. $\frac{93}{154}$ Q15. $\frac{17}{400}$ Q16. 0.26 Q17. $\frac{13}{150}$ Q18. $\frac{32}{55}$

Q19. 0.488

EXERCISE 13.4

Q01. $\frac{2}{5}$ Q02. $\frac{25}{69}, \frac{28}{69}, \frac{16}{69}$ Q03. $\frac{11}{31}$ Q04. $\frac{27}{83}$ Q05. $\frac{6}{52}$

Q06. $\frac{3}{8}$ Q07. $\frac{4}{9}$ Q08. $\frac{24}{29}$ Q09. $\frac{11}{50}$ Q10. $\frac{2}{3}$

Q11. $\frac{20}{21}$ Q12. $\frac{5}{41}, \frac{36}{41}$ Q13. 0.083 *i.e.*, 8.3% (approx.) Q14. $\frac{110}{221}$

Q15. $\frac{14}{29}$. **Interpretation of result:** It is evident that if a patient follows a course of meditation and yoga, then he is less likely to get heart-attack. Since $P(B|E)=\frac{15}{29}$. So, a course of meditation and yoga is more beneficial as compared to the intake of drugs.

Q16. $\frac{22}{133}$ Q17. $\frac{1}{2}$ Q18. $\frac{12}{13}$ Q19. $\frac{4}{9}$ Q20. $\frac{8}{11}$ Q21. 0.95 Q22. $\frac{16}{31}$

Q23. $\frac{10}{43}$ Q24. $\frac{20}{37}$ Q25. $\frac{5}{13}$ Q26. $\frac{3}{50}$ Q27. $\frac{7}{11}$ Q28. $\frac{9}{13}$ Q29. $\frac{10}{19}$

Q30. $\frac{2}{9}$ Q31. $\frac{8}{9}$ Q32. $\frac{7}{10}$ Q33. $\frac{3}{5}$ Q34. $\frac{9}{17}$ Q35. $\frac{3}{5}$ Q36. $\frac{56}{117}$

Q37. $\frac{4}{35}$

EXERCISE 13.5

Q01. $\frac{3}{16}$ Q02. $P(\text{A wins})=\frac{6}{11}$ and, $P(\text{B wins})=\frac{5}{11}$ Q03. $P(\text{A wins})=\frac{30}{61}, P(\text{B wins})=\frac{31}{61}$

Q04. $P(\text{A wins})=\frac{36}{91}$, $P(\text{B wins})=\frac{30}{91}$ and $P(\text{C wins})=\frac{25}{91}$

Q05. $\frac{4}{7}$ Q06. $\frac{81}{217}, \frac{72}{217}, \frac{64}{217}$ Q07. $\frac{5}{17}$ Q08. $\frac{2}{30}$

Q09. $\frac{1}{3}$ Q10. (a) $\frac{1}{15}$ (b) $\frac{6}{15}$ (c) $\frac{8}{15}$ Q11. 0.165 Q12. $\frac{4}{63}$

Q13. $\frac{2}{3}$ Q14. $\frac{1}{17}$ Q15. $\frac{22}{45}$ Q16. $\frac{28}{45}$

Q17. (i) 0.04 (ii) 0.74.

ANSWER KEYS (Multiple Choice type Questions)

Chapter 01

Q01.	(d)	Q02.	(b)	Q03.	(a)	Q04.	(a)	Q05.	(c)	Q06.	(c)	Q07.	(a)
Q08.	(a)	Q09.	(b)	Q10.	(d)	Q11.	(b)	Q12.	(b)	Q13.	(b)	Q14.	(c)
Q15.	(b)	Q16.	(b)	Q17.	(a)	Q18.	(a)	Q19.	(c)	Q20.	(b)	Q21.	(b)
Q22.	(c)	Q23.	(b)	Q24.	(d)	Q25.	(d)	Q26.	(d)	Q27.	(d)	Q28.	(c)
Q29.	(b)	Q30.	(d)	Q31.	(d)	Q32.	(d)	Q33.	(c)	Q34.	(b)	Q35.	(d)
Q36.	(b)	Q37.	(a)	Q38.	(a)	Q39.	(a)	Q40.	(d)	Q41.	(a)	Q42.	(c)
Q43.	(a)	Q44.	(c)	Q45.	(c)	Q46.	(c)	Q47.	(d)	Q48.	(a)	Q49.	(c)
Q50.	(c)	Q51.	(c)	Q52.	(d)	Q53.	(c)	Q54.	(c)	Q55.	(c)	Q56.	(b)
Q57.	(b)	Q58.	(a)	Q59.	(b)	Q60.	(c)	Q61.	(b)	Q62.	(c)	Q63.	(b)
Q64.	(c)	Q65.	(a)	Q66.	(a)	Q67.	(a)	Q68.	(b)	Q69.	(b)	Q70.	(b)
Q71.	(a)	Q72.	(b)	Q73.	(d)	Q74.	(a)	Q75.	(a)	Q76.	(a)	Q77.	(b)
Q78.	(b)	Q79.	(a)	Q80.	(c)	Q81.	(b)	Q82.	(a)	Q83.	(b)	Q84.	(d)
Q85.	(a)	Q86.	(d)	Q87.	(b)	Q88.	(a)	Q89.	(c)	Q90.	(b)	Q91.	(b)
Q92.	(c)	Q93.	(d)	Q94.	(a)	Q95.	(b)	Q96.	(c)	Q97.	(d)	Q98.	(d)
Q99.	(c)	Q100.	(a)	Q101.	(d)	Q102.	(d)	Q103.	(b)	Q104.	(c)	Q105.	(b)
Q106.	(b)	Q107.	(c)	Q108.	(b)	Q109.	(d)	Q110.	(d)	Q111.	(b)	Q112.	(d)
Q113.	(b)	Q114.	(d)	Q115.	(b)	Q116.	(a)	Q117.	(c)	Q118.	(d)	Q119.	(b)
Q120.	(b)	Q121.	(d)	Q122.	(a)	Q123.	(b)	Q124.	(d)	Q125.	(d)	Q126.	(d)
Q127.	(b)	Q128.	(c)	Q129.	(b)	Q130.	(c)	Q131.	(a)	Q132.	(b)	Q133.	(c)
Q134.	(a)	Q135.	(c)	Q136.	(b)	Q137.	(d)	Q138.	(c)	Q139.	(c)	Q140.	(a)
Q141.	(a)	Q142.	(d)	Q143.	(a)	Q144.	(d)	Q145.	(a)	Q146.	(a)	Q147.	(b)
Q148.	(d)	Q149.	(b)	Q150.	(d)	Q151.	(d)	Q152.	(d)	Q153.	(b)	Q154.	(a)
Q155.	(d)	Q156.	(c)	Q157.	(d)	Q158.	(c)						

Chapter 02

Q01.	(c)	Q02.	(c)	Q03.	(a)	Q04.	(c)	Q05.	(b)	Q06.	(d)	Q07.	(a)
Q08.	(b)	Q09.	(c)	Q10.	(b)	Q11.	(c)	Q12.	(b)	Q13.	(b)	Q14.	(a)
Q15.	(a)	Q16.	(a)	Q17.	(b)	Q18.	(d)	Q19.	(d)	Q20.	(b)	Q21.	(c)
Q22.	(b)	Q23.	(d)	Q24.	(b)	Q25.	(d)	Q26.	(b)	Q27.	(a)	Q28.	(b)
Q29.	(a)	Q30.	(b)	Q31.	(b)	Q32.	(c)	Q33.	(a)	Q34.	(b)	Q35.	(a)
Q36.	(c)	Q37.	(d)	Q38.	(d)	Q39.	(d)	Q40.	(b)	Q41.	(a)	Q42.	(c)
Q43.	(a)	Q44.	(c)	Q45.	(b)	Q46.	(d)						

Chapter 03

Q01.	(b)	Q02.	(d)	Q03.	(a)	Q04.	(a)	Q05.	(d)	Q06.	(d)	Q07.	(d)
Q08.	(b)	Q09.	(a)	Q10.	(b)	Q11.	(d)	Q12.	(a)	Q13.	(a)	Q14.	(b)
Q15.	(c)	Q16.	(d)	Q17.	(d)	Q18.	(a)	Q19.	(d)	Q20.	(b)	Q21.	(c)
Q22.	(c)	Q23.	(a)	Q24.	(b)	Q25.	(a)	Q26.	(b)	Q27.	(b)	Q28.	(d)
Q29.	(b)	Q30.	(d)	Q31.	(c)	Q32.	(d)	Q33.	(c)	Q34.	(a)	Q35.	(b)
Q36.	(c)	Q37.	(b)	Q38.	(d)	Q39.	(a)	Q40.	(b)	Q41.	(c)		

Chapter 04

Q01.	(a)	Q02.	(a)	Q03.	(b)	Q04.	(d)	Q05.	(a)	Q06.	(c)	Q07.	(b)
Q08.	(a)	Q09.	(a)	Q10.	(c)	Q11.	(a)	Q12.	(d)	Q13.	(a)	Q14.	(a)
Q15.	(c)	Q16.	(b)	Q17.	(d)	Q18.	(c)	Q19.	(c)	Q20.	(a)	Q21.	(b)
Q22.	(d)	Q23.	(d)	Q24.	(c)	Q25.	(d)	Q26.	(b)	Q27.	(c)	Q28.	(b)
Q29.	(c)	Q30.	(d)	Q31.	(a)	Q32.	(c)	Q33.	(d)	Q34.	(d)	Q35.	(a)
Q36.	(a)	Q37.	(a)	Q38.	(c)	Q39.	(d)	Q40.	(d)	Q41.	(b)	Q42.	(d)
Q43.	(b)	Q44.	(b)	Q45.	(a)	Q46.	(c)	Q47.	(c)	Q48.	(b)	Q49.	(c)
Q50.	(b)	Q51.	(d)	Q52.	(b)	Q53.	(d)	Q54.	(a)	Q55.	(b)	Q56.	(d)
Q57.	(a)	Q58.	(c)	Q59.	(d)	Q60.	(b)	Q61.	(d)	Q62.	(a)	Q63.	(a)
Q64.	(b)	Q65.	(c)	Q66.	(c)	Q67.	(d)	Q68.	(c)	Q69.	(c)	Q70.	(c)
Q71.	(b)	Q72.	(b)	Q73.	(a)	Q74.	(a)	Q75.	(c)	Q76.	(d)	Q77.	(d)
Q78.	(a)	Q79.	(c)										

Chapter 05

Q01.	(a)	Q02.	(a)	Q03.	(a)	Q04.	(d)	Q05.	(a)	Q06.	(a)	Q07.	(a)
Q08.	(b)	Q09.	(c)	Q10.	(b)	Q11.	(d)	Q12.	(a)	Q13.	(c)	Q14.	(a)
Q15.	(c)	Q16.	(b)	Q17.	(c)	Q18.	(b)	Q19.	(b)	Q20.	(b)	Q21.	(d)

Q22.	(d)	**Q23.**	(c)	**Q24.**	(a)	**Q25.**	(d)	**Q26.**	(d)	**Q27.**	(a)	**Q28.**	(a)
Q29.	(c)	**Q30.**	(a)	**Q31.**	(d)	**Q32.**	(c)	**Q33.**	(c)	**Q34.**	(c)	**Q35.**	(c)
Q36.	(a)	**Q37.**	(d)	**Q38.**	(a)	**Q39.**	(d)	**Q40.**	(d)	**Q41.**	(b)	**Q42.**	(b)
Q43.	(a)	**Q44.**	(a)	**Q45.**	(c)	**Q46.**	(c)	**Q47.**	(a)	**Q48.**	(c)	**Q49.**	(c)
Q50.	(c)	**Q51.**	(b)	**Q52.**	(b)	**Q53.**	(b)	**Q54.**	(b)	**Q55.**	(c)	**Q56.**	(c)
Q57.	(a)	**Q58.**	(a)	**Q59.**	(b)								

Chapter 06

Q01.	(b)	**Q02.**	(c)	**Q03.**	(c)	**Q04.**	(c)	**Q05.**	(b)	**Q06.**	(a)	**Q07.**	(d)
Q08.	(c)	**Q09.**	(c)	**Q10.**	(b)	**Q11.**	(c)	**Q12.**	(d)	**Q13.**	(a)	**Q14.**	(b)
Q15.	(d)	**Q16.**	(d)	**Q17.**	(d)	**Q18.**	(d)	**Q19.**	(c)	**Q20.**	(a)	**Q21.**	(a)
Q22.	(c)	**Q23.**	(a)	**Q24.**	(c)	**Q25.**	(c)	**Q26.**	(d)	**Q27.**	(d)	**Q28.**	(a)
Q29.	(c)	**Q30.**	(a)	**Q31.**	(b)	**Q32.**	(b)	**Q33.**	(d)	**Q34.**	(c)	**Q35.**	(a)
Q36.	(c)	**Q37.**	(c)	**Q38.**	(c)	**Q39.**	(c)	**Q40.**	(b)	**Q41.**	(b)	**Q42.**	(b)
Q43.	(c)	**Q44.**	(b)	**Q45.**	(b)	**Q46.**	(b)	**Q47.**	(a)				

Chapter 07

Q01.	(a)	**Q02.**	(b)	**Q03.**	(d)	**Q04.**	(a)	**Q05.**	(d)	**Q06.**	(c)	**Q07.**	(d)
Q08.	(d)	**Q09.**	(c)	**Q10.**	(c)	**Q11.**	(d)	**Q12.**	(a)	**Q13.**	(d)	**Q14.**	(d)
Q15.	(b)	**Q16.**	(b)	**Q17.**	(c)	**Q18.**	(b)	**Q19.**	(b)	**Q20.**	(b)	**Q21.**	(a)
Q22.	(b)	**Q23.**	(c)	**Q24.**	(c)	**Q25.**	(b)	**Q26.**	(b)	**Q27.**	(d)	**Q28.**	(d)
Q29.	(b)	**Q30.**	(a)	**Q31.**	(b)	**Q32.**	(d)	**Q33.**	(b)	**Q34.**	(d)	**Q35.**	(c)
Q36.	(b)	**Q37.**	(c)	**Q38.**	(a)	**Q39.**	(a)	**Q40.**	(a)	**Q41.**	(d)	**Q42.**	(b)
Q43.	(c)	**Q44.**	(d)	**Q45.**	(a)	**Q46.**	(b)	**Q47.**	(d)	**Q48.**	(b)		

Chapter 08

Q01.	(d)	**Q02.**	(b)	**Q03.**	(b)	**Q04.**	(d)	**Q05.**	(a)	**Q06.**	(c)	**Q07.**	(b)
Q08.	(b)	**Q09.**	(c)	**Q10.**	(b)	**Q11.**	(a)	**Q12.**	(d)	**Q13.**	(a)	**Q14.**	(c)
Q15.	(b)	**Q16.**	(d)	**Q17.**	(a)	**Q18.**	(b)	**Q19.**	(d)	**Q20.**	(c)	**Q21.**	(b)
Q22.	(a)	**Q23.**	(a)	**Q24.**	(d)	**Q25.**	(b)	**Q26.**	(b)	**Q27.**	(b)	**Q28.**	(d)
Q29.	(d)	**Q30.**	(c)	**Q31.**	(b)	**Q32.**	(a)	**Q33.**	(a)				

Chapter 09

Q01.	(b)	**Q02.**	(a)	**Q03.**	(b)	**Q04.**	(b)	**Q05.**	(d)	**Q06.**	(c)	**Q07.**	(c)
Q08.	(c)	**Q09.**	(b)	**Q10.**	(c)	**Q11.**	(d)	**Q12.**	(c)	**Q13.**	(a)	**Q14.**	(c)
Q15.	(c)	**Q16.**	(c)	**Q17.**	(b)	**Q18.**	(d)	**Q19.**	(d)	**Q20.**	(d)	**Q21.**	(d)
Q22.	(d)	**Q23.**	(b)	**Q24.**	(c)	**Q25.**	(c)	**Q26.**	(a)	**Q27.**	(b)	**Q28.**	(c)
Q29.	(c)	**Q30.**	(d)	**Q31.**	(c)	**Q32.**	(c)	**Q33.**	(d)	**Q34.**	(d)	**Q35.**	(a)
Q36.	(d)	**Q37.**	(a)	**Q38.**	(d)	**Q39.**	(b)	**Q40.**	(c)	**Q41.**	(b)	**Q42.**	(c)

Chapter 10

Q01.	(d)	**Q02.**	(b)	**Q03.**	(d)	**Q04.**	(b)	**Q05.**	(c)	**Q06.**	(d)	**Q07.**	(a)
Q08.	(c)	**Q09.**	(c)	**Q10.**	(b)	**Q11.**	(a)	**Q12.**	(b)	**Q13.**	(a)	**Q14.**	(a)
Q15.	(a)	**Q16.**	(d)	**Q17.**	(a)	**Q18.**	(d)	**Q19.**	(b)	**Q20.**	(c)	**Q21.**	(c)
Q22.	(a)	**Q23.**	(c)	**Q24.**	(a)	**Q25.**	(b)	**Q26.**	(b)	**Q27.**	(d)	**Q28.**	(c)
Q29.	(b)	**Q30.**	(a)	**Q31.**	(d)	**Q32.**	(b)	**Q33.**	(a)	**Q34.**	(b)	**Q35.**	(c)
Q36.	(a)	**Q37.**	(c)	**Q38.**	(c)	**Q39.**	(d)	**Q40.**	(b)	**Q41.**	(c)	**Q42.**	(a)
Q43.	(c)	**Q44.**	(d)	**Q45.**	(d)	**Q46.**	(c)	**Q47.**	(c)	**Q48.**	(d)	**Q49.**	(b)
Q50.	(c)	**Q51.**	(d)	**Q52.**	(b)	**Q53.**	(c)						

Chapter 11

Q01.	(b)	**Q02.**	(d)	**Q03.**	(a)	**Q04.**	(b)	**Q05.**	(d)	**Q06.**	(b)	**Q07.**	(c)
Q08.	(a)	**Q09.**	(a)	**Q10.**	(a)	**Q11.**	(c)	**Q12.**	(d)	**Q13.**	(c)	**Q14.**	(b)
Q15.	(d)	**Q16.**	(a)	**Q17.**	(d)	**Q18.**	(d)	**Q19.**	(c)	**Q20.**	(a)	**Q21.**	(c)
Q22.	(c)	**Q23.**	(b)	**Q24.**	(a)	**Q25.**	(b)	**Q26.**	(d)	**Q27.**	(a)	**Q28.**	(b)
Q29.	(c)	**Q30.**	(d)	**Q31.**	(a)	**Q32.**	(b)	**Q33.**	(d)	**Q34.**	(d)	**Q35.**	(c)
Q36.	(b)	**Q37.**	(c)	**Q38.**	(d)	**Q39.**	(d)	**Q40.**	(a)	**Q41.**	(b)	**Q42.**	(b)
Q43.	(a)	**Q44.**	(d)	**Q45.**	(c)	**Q46.**	(a)	**Q47.**	(a)	**Q48.**	(a)	**Q49.**	(a)
Q50.	(a)	**Q51.**	(d)	**Q52.**	(b)	**Q53.**	(d)	**Q54.**	(b)	**Q55.**	(b)	**Q56.**	(b)
Q57.	(b)	**Q58.**	(c)	**Q59.**	(c)	**Q60.**	(b)	**Q61.**	(a)	**Q62.**	(b)	**Q63.**	(b)
Q64.	(a)	**Q65.**	(d)	**Q66.**	(c)	**Q67.**	(a)	**Q68.**	(c)	**Q69.**	(c)	**Q70.**	(d)
Q71.	(c)	**Q72.**	(d)	**Q73.**	(b)	**Q74.**	(d)						

Chapter 12

Q01.	(d)	**Q02.**	(d)	**Q03.**	(a)	**Q04.**	(a)	**Q05.**	(a)	**Q06.**	(b)	**Q07.**	(c)
Q08.	(d)	**Q09.**	(b)	**Q10.**	(b)	**Q11.**	(b)	**Q12.**	(d)	**Q13.**	(a)	**Q14.**	(c)
Q15.	(b)	**Q16.**	(b)	**Q17.**	(c)	**Q18.**	(d)	**Q19.**	(d)	**Q20.**	(a)	**Q21.**	(a)
Q22.	(b)	**Q23.**	(a)	**Q24.**	(b)	**Q25.**	(a)	**Q26.**	(c)	**Q27.**	(a)	**Q28.**	(d)
Q29.	(c)	**Q30.**	(a)	**Q31.**	(d)	**Q32.**	(d)	**Q33.**	(d)	**Q34.**	(d)	**Q35.**	(d)
Q36.	(d)	**Q37.**	(d)	**Q38.**	(b)	**Q39.**	(b)	**Q40.**	(b)	**Q41.**	(d)		

Chapter 13

Q01.	(a)	**Q02.**	(b)	**Q03.**	(d)	**Q04.**	(c)	**Q05.**	(d)	**Q06.**	(a)	**Q07.**	(a)
Q08.	(a)	**Q09.**	(d)	**Q10.**	(c)	**Q11.**	(c)	**Q12.**	(d)	**Q13.**	(c)	**Q14.**	(d)
Q15.	(c)	**Q16.**	(d)	**Q17.**	(d)	**Q18.**	(d)	**Q19.**	(c)	**Q20.**	(d)	**Q21.**	(d)
Q22.	(c)	**Q23.**	(c)	**Q24.**	(c)	**Q25.**	(b)	**Q26.**	(b)	**Q27.**	(d)	**Q28.**	(c)
Q29.	(a)	**Q30.**	(d)	**Q31.**	(c)	**Q32.**	(a)	**Q33.**	(d)	**Q34.**	(b)	**Q35.**	(a)
Q36.	(c)	**Q37.**	(a)	**Q38.**	(c)	**Q39.**	(d)	**Q40.**	(c)	**Q41.**	(c)	**Q42.**	(a)
Q43.	(d)	**Q44.**	(d)	**Q45.**	(a)	**Q46.**	(a)	**Q47.**	(c)	**Q48.**	(b)	**Q49.**	(d)
Q50.	(d)	**Q51.**	(c)	**Q52.**	(c)	**Q53.**	(c)	**Q54.**	(c)	**Q55.**	(c)	**Q56.**	(d)
Q57.	(c)	**Q58.**	(b)	**Q59.**	(c)	**Q60.**	(c)	**Q61.**	(b)	**Q62.**	(c)	**Q63.**	(d)

💻 ANSWER KEYS (Assertion-Reason type Questions)

Unit 1 (Relations & Functions)

Q01.	(a)	Q02.	(d)	Q03.	(d)	Q04.	(a)	Q05.	(a)	Q06.	(d)	Q07.	(a)
Q08.	(d)	Q09.	(a)	Q10.	(d)	Q11.	(a)	Q12.	(a)	Q13.	(d)	Q14.	(a)
Q15.	(a)	Q16.	(d)	Q17.	(b)	Q18.	(a)	Q19.	(a)	Q20.	(a)	Q21.	(d)
Q22.	(a)	Q23.	(b)	Q24.	(a)	Q25.	(a)	Q26.	(c)	Q27.	(a)	Q28.	(c)
Q29.	(b)	Q30.	(d)	Q31.	(d)	Q32.	(d)	Q33.	(c)	Q34.	(c)	Q35.	(c)
Q36.	(d)	Q37.	(c)	Q38.	(d)	Q39.	(c)	Q40.	(c)				

Unit 2 (Algebra)

Q01.	(b)	Q02.	(c)	Q03.	(b)	Q04.	(b)	Q05.	(b)	Q06.	(d)	Q07.	(a)
Q08.	(a)	Q09.	(b)	Q10.	(a)	Q11.	(a)	Q12.	(b)	Q13.	(c)	Q14.	(a)
Q15.	(a)	Q16.	(c)	Q17.	(a)	Q18.	(b)	Q19.	(d)	Q20.	(a)	Q21.	(d)
Q22.	(a)	Q23.	(a)	Q24.	(a)	Q25.	(c)	Q26.	(b)	Q27.	(a)	Q28.	(a)
Q29.	(a)	Q30.	(b)	Q31.	(c)	Q32.	(b)	Q33.	(a)	Q34.	(a)	Q35.	(c)
Q36.	(d)	Q37.	(a)	Q38.	(a)	Q39.	(d)	Q40.	(c)				

Unit 3 (Calculus)

Q01.	(d)	Q02.	(c)	Q03.	(d)	Q04.	(a)	Q05.	(c)	Q06.	(d)	Q07.	(a)
Q08.	(a)	Q09.	(c)	Q10.	(a)	Q11.	(c)	Q12.	(d)	Q13.	(d)	Q14.	(a)
Q15.	(b)	Q16.	(d)	Q17.	(d)	Q18.	(a)	Q19.	(a)	Q20.	(b)	Q21.	(c)
Q22.	(c)	Q23.	(d)	Q24.	(a)	Q25.	(a)	Q26.	(c)	Q27.	(d)	Q28.	(d)
Q29.	(d)	Q30.	(c)	Q31.	(d)	Q32.	(c)	Q33.	(b)	Q34.	(a)	Q35.	(c)
Q36.	(a)	Q37.	(b)	Q38.	(a)	Q39.	(a)	Q40.	(b)	Q41.	(b)	Q42.	(a)
Q43.	(b)	Q44.	(b)	Q45.	(a)	Q46.	(b)	Q47.	(b)	Q48.	(a)	Q49.	(a)
Q50.	(b)	Q51.	(c)	Q52.	(c)	Q53.	(d)	Q54.	(a)	Q55.	(b)	Q56.	(a)
Q57.	(a)	Q58.	(a)	Q59.	(d)	Q60.	(a)	Q61.	(a)	Q62.	(d)		

Unit 4 (Vectors & 3 D Geometry)

Q01.	(b)	Q02.	(a)	Q03.	(a)	Q04.	(b)	Q05.	(a)	Q06.	(c)	Q07.	(a)
Q08.	(a)	Q09.	(c)	Q10.	(a)	Q11.	(d)	Q12.	(a)	Q13.	(d)	Q14.	(c)
Q15.	(d)	Q16.	(a)	Q17.	(c)	Q18.	(a)	Q19.	(c)	Q20.	(a)	Q21.	(a)
Q22.	(b)	Q23.	(a)	Q24.	(b)	Q25.	(a)	Q26.	(b)	Q27.	(a)	Q28.	(b)
Q29.	(b)	Q30.	(b)	Q31.	(d)	Q32.	(a)	Q33.	(c)	Q34.	(a)	Q35.	(d)
Q36.	(a)	Q37.	(c)	Q38.	(d)	Q39.	(d)	Q40.	(c)	Q41.	(b)	Q42.	(a)
Q43.	(d)	Q44.	(c)	Q45.	(d)								

Unit 5 (Linear Programming)

Q01.	(a)	Q02.	(c)	Q03.	(d)	Q04.	(d)	Q05.	(b)	Q06.	(a)	Q07.	(d)
Q08.	(c)	Q09.	(c)	Q10.	(d)	Q11.	(b)	Q12.	(c)				

Unit 6 (Probability)

Q01.	(a)	Q02.	(a)	Q03.	(d)	Q04.	(c)	Q05.	(c)	Q06.	(b)	Q07.	(a)
Q08.	(d)	Q09.	(c)	Q10.	(c)	Q11.	(b)	Q12.	(a)	Q13.	(a)	Q14.	(a)
Q15.	(d)	Q16.	(a)	Q17.	(b)	Q18.	(d)	Q19.	(c)	Q20.	(c)	Q21.	(d)
Q22.	(a)	Q23.	(d)										

ANSWERS OF CASE STUDY QUESTIONS

Unit I - Relations & Functions

Q01. **(i)** 2^{20}. **(ii)** R is equivalence relation. **(iii)** $2^{10}(31)$.
(iv) R is not a function. **(v)** 240.

Q02. **(i)** $y=\frac{1}{x^2},\ x\neq 0$. **(ii)** $x\in(0,\infty),\ y\in(0,\infty)$.
(iii) As $f(\alpha)=f(\beta)$ implies $\alpha=\beta$, so, $y=f(x)=\frac{1}{x^2},\ x\neq 0$ is a one-one function.
(iv) $\frac{3}{16}$.

Q03. **(i)** R, [–1, 1].
(ii) When suitable restriction is imposed on the domain of sine function, it becomes invertible. Therefore, the sine function becomes one-one and onto both.
(iii) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. **(iv)** $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$. **(v)** $-\frac{\pi}{4}$.

Q04. **(i)** Transitive only. **(ii)** Transitive but not symmetric.
(iii) Reflexive and identity, both. **(iv)** 2^{12}. **(v)** $2^6=64$.

Q05. **(i)** Transitive only. **(ii)** Transitive but not symmetric.
(iii) Reflexive and identity, both. **(iv)** 2^{10}. **(v)** $2^6=64$.

Q06. **(i)** $\frac{\pi}{4}$. **(ii)** $20\sqrt{2}$ m. **(iii)** $\tan^{-1}(2)$ **(iv)** $\frac{3\pi}{4}-\cot^{-1}\frac{1}{2}$

Q07. **(i)** $(X, Y)\in R$ is true. **(ii)** Given statement is not true. **(iii)** R is symmetric.
(iv) We can not have $(\text{Ghanshyam, Radheshyam})\in R$, $(\text{Radhika, Radheshyam})\notin R$ is a correct statement.

Q08. **(i)** 64. **(ii)** 8. **(iii)** R is an equivalence relation.
(iv) f is not bijective.

Q09. **(i)** R is symmetric. **(ii)** R is transitive.
(iii) Set of all lines related to the given line is given by $y=3x+c,\ c\in\mathbb{R}$.
(iv) The relation S is symmetric, but relation S is not transitive.

Q10. **(i)** $A=\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ **(ii)** $-\frac{2\pi}{3}$
(iii) See the graph in the Theory section of this Chapter.
(iv) Domain : $x\in[0,2]$; range : $[-\pi,\pi]$.

Q11. **(i)** (a) **(ii)** (d) **(iii)** (c) **(iv)** (c) **(v)** (a)
Q12. **(i)** (c) **(ii)** (c) **(iii)** (a) **(iv)** (d) **(v)** (b)

Q13. **(i)** (b) **(ii)** (c) **(iii)** (d) **(iv)** (b) **(v)** (a)

Q14. **(i)** 64 **(ii)** $\{(g_1, g_1), (g_2, g_2)\}$

(iii) (A) $(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)$.

Note that, it can be any one of the pair from, $(b_3, b_2), (b_1, b_3), (b_3, b_1)$ in place of the pair (b_2, b_3) also.

(B) $(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)$.

(iv) f is one-one and onto.

Q15. **(i)** R is reflexive relation **(ii)** R is symmetric.

(iii) R is transitive; R is an equivalence relation. **(iv)** f is not one-one; f is onto.

Q16. **(i)** $\theta = \tan^{-1}\frac{80}{x}$ **(ii)** $\frac{\pi}{4}$

(iii) $x = 60$ m **(iv)** $\tan^{-1}\frac{2}{3} \le \theta \le \tan^{-1}\frac{4}{3}$

Unit II - Algebra

Q01. **(i)** Combined Sale in September and October,

$$A+B = \begin{pmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{pmatrix} \begin{matrix} \text{Ramkrishna} \\ \text{Hari Prasad} \end{matrix}$$

(columns: Basmati, Permal, Naura)

(ii) Decrease in Sales in September to October,

$$A-B = \begin{pmatrix} 5000 & 10000 & 24000 \\ 30000 & 20000 & 0 \end{pmatrix} \begin{matrix} \text{Ramkrishna} \\ \text{Hari Prasad} \end{matrix}$$

(columns: Basmati, Permal, Naura)

(iii) ₹800. **(iv)** ₹420. **(v)** ₹600.

Q02. **(i)** $\begin{pmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11000 \\ 10700 \\ 2700 \end{pmatrix}$.

(ii) $5x+4y+3z=11000,\ 4x+3y+5z=10700,\ x+y+z=2700$.

(iii) Consistent system with unique solution. **(iv)** ₹1000. **(v)** ₹4600.

Q03. **(i)** $20x+5y=9000$ and $5x+25y=26000$. **(ii)** $\begin{pmatrix} 20 & 5 \\ 5 & 25 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9000 \\ 26000 \end{pmatrix}$.

(iii) ₹200. **(iv)** ₹1000. **(v)** ₹800.

Q04. **(i)** ₹7000. **(ii)** ₹14000. **(iii)** ₹21000.

(iv) ₹21250. **(v)** 330.

Q05. **(i)** $x+y+z=7000,\ x-y=0,\ 10x+16y+17z=110000$.

(ii) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 10 & 16 & 17 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7000 \\ 0 \\ 110000 \end{pmatrix}$. **(iii)** $2x+z=7000,\ 26x+17z=110000$.

(iv) ₹1125. **(v)** ₹4750.

Q06. **(i)** $AX = B$; where $A = \begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}, B = \begin{pmatrix} 145 \\ 180 \end{pmatrix}, X = \begin{pmatrix} x \\ y \end{pmatrix}$.

(ii) 91. **(iii)** ₹10. **(iv)** ₹15. **(v)** ₹65.

Q07. **(i)** $x+y+z=21,\ 4x+3y+2z=60,\ 6x+2y+3z=70$.

(ii) $AX=B$ where $A=\begin{pmatrix}1&1&1\\4&3&2\\6&2&3\end{pmatrix}$, $X=\begin{pmatrix}x\\y\\z\end{pmatrix}$ and $B=\begin{pmatrix}21\\60\\70\end{pmatrix}$.

(iii) -5. **(iv)** 4. **(v)** ₹5.

Q08. **(i)** $x-y=50,\ x+2y=500$. **(ii)** $\begin{pmatrix}1&-1\\2&1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}50\\550\end{pmatrix}$.

(iii) Dimensions of the land are : length $=200$ m and breadth $=150$ m.

Area of land $=30000\ m^2$. **(iv)** $x=200$ m, $y=150$ m.

Q09. **(i)** $\begin{pmatrix}5&3&1\\2&1&3\\1&2&4\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}160\\190\\250\end{pmatrix}$. **(ii)** -22.

(iii) $\frac{1}{22}\begin{pmatrix}2&10&-8\\5&-19&13\\-3&7&1\end{pmatrix}$. **(iv)** $\begin{pmatrix}7&5&13\\5&8&2\\8&3&3\end{pmatrix}$.

Q10. **(i)** $\begin{bmatrix}3&4\\1&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}180\\50\end{bmatrix}$

(ii) System of matrix equations so obtained is consistent.

(iii) No. of scholarships given to girl students is 20 and that of meritorious achievers is 30.

(iv) ₹170000.

Q11. **(i)** (a) **(ii)** (c) **(iii)** (d) **(iv)** (b) **(v)** (c)

Q12. **(i)** (b) **(ii)** (a) **(iii)** (c) **(iv)** (b) **(v)** (c)

Q13. **(i)** (b) **(ii)** (b) **(iii)** (a) **(iv)** (c) **(v)** (c)

Unit III - Calculus

Q01. **(i)** $xy=150$. **(ii)** $(x-3)(y-2)$. **(iii)** $156-2x-3\left(\frac{150}{x}\right)$.

(iv) $x=15$ cm. **(v)** $x=15$ cm, $y=10$ cm.

Q02. **(i)** $4y=10-(\pi+2)x$ **(ii)** $A=10x-\left(2+\frac{1}{2}\pi\right)x^2$ **(iii)** $A=\frac{50}{\pi+4}\ m^2$.

(iv) $\frac{20}{\pi+4}$ m. **(v)** $\frac{50\pi}{(\pi+4)^2}\ m^2$.

Q03. **(i)** $x^2+4xy=C$. **(ii)** $y=\frac{C-x^2}{4x}$. **(iii)** $\frac{Cx-x^3}{4}$. **(iv)** $x=\sqrt{\frac{C}{3}}$ units.

(v) $\frac{C}{6}\times\sqrt{\frac{C}{3}}$ units^3.

Q04. **(i)** $y^2+4xy=C^2$. **(ii)** $x=\frac{C^2-y^2}{4y}$. **(iii)** $\frac{C^2y-y^3}{4}$

	(iv)	$y=\frac{C}{\sqrt{3}}$ units.	**(v)**	$\frac{C^3}{6\sqrt{3}}$ units^3.		
Q05.	**(i)**	$3x^2-90x+600$.	**(ii)**	$x=10, 20$.	**(iii)**	20 trees.
	(iv)	₹2000.	**(v)**	₹2500.		
Q06.	**(i)**	$2x^2-40x+1140$.	**(ii)**	$4x-40$.	**(iii)**	$x=10$ m.
	(iv)	$x=15$ m.	**(v)**	10 m.		
Q07.	**(i)**	$f(x)=24x-22x^2+4x^3$.			**(ii)**	$f'(x)=24-44x+12x^2$.
	(iii)	$x=\frac{2}{3}$ m.	**(iv)**	$\frac{20}{3}$ m.	**(v)**	$\frac{200}{27}$ m^3.
Q08.	**(i)**	x^2+4hx.	**(ii)**	$4000=x^2h$.	**(iii)**	$2x-\frac{16000}{x^2}$.
	(iv)	$x=20$ m.	**(v)**	1200 m^2.		
Q09.	**(i)**	₹$(250x-x^2)$.	**(ii)**	$300-6x$.	**(iii)**	50 bulbs.
	(iv)	₹7488.	**(v)**	100 bulbs.		
Q10.	**(i)**	$150000+200x-x^2$.	**(ii)**	$200-2x$.	**(iii)**	-2.
	(iv)	₹100.	**(v)**	₹10000.		
Q11.	**(i)**	$h=\frac{v}{\pi r^2}$.	**(ii)**	$\pi r^2+\frac{2v}{r}$.	**(iii)**	$r=\left(\frac{v}{\pi}\right)^{1/3}$.
	(iv)	$h=\left(\frac{v}{\pi}\right)^{1/3}$.	**(v)**	$r=h$.		
Q12.	**(i)**	$A=\left(\frac{40-x}{4}\right)^2+\frac{x^2}{12\sqrt{3}}$.	**(ii)**	$\frac{x}{6\sqrt{3}}+\left(\frac{x-40}{8}\right)$.	**(iii)**	$\frac{120\sqrt{3}}{4+3\sqrt{3}}$ m.
	(iv)	$\frac{160}{4+3\sqrt{3}}$ m.	**(v)**	$\frac{400}{4+3\sqrt{3}}$ m^2.		
Q13.	**(i)**	$f(x)=\sqrt{(x-3)^2+x^4}$	**(ii)**	$2(x-3)+4x^3$	**(iii)**	$2+12x^2$.
	(iv)	$\sqrt{5}$ units.	**(v)**	(1, 8).		
Q14.	**(i)**	$R=R_1-\frac{R_1^2}{K}$.	**(ii)**	$1-\frac{2R_1}{K}$.	**(iii)**	$R_1=\frac{K}{2}$.
	(iv)	$R_1=R_2$.	**(v)**	$R=\frac{K}{4}$.		
Q15.	**(i)**	$2x+2y+\pi y=1000$.	**(ii)**	$\frac{2}{2+\pi}(1000x-2x^2)$.	**(iii)**	$x=250$ m.
	(iv)	$x=\frac{1000}{\pi+4}$ m.	**(v)**	$A=\frac{250000}{2+\pi}$ m^2.		
Q16.	**(i)**	$V=x^2y$.	**(ii)**	$C=$ ₹$5(3x^2+4xy)$.		
	(iii)	$C=$ ₹$5\left(3x^2+\frac{4V}{x}\right)$.	**(iv)**	$x=8$ m, $y=12$ m.	**(v)**	₹2880.

Q17. **(i)** $h=\frac{1000}{r^2}$. **(ii)** $S=\pi r^2+\frac{2000\pi}{r}$. **(iii)** $r=10$ m.

(iv) $h=10$ m. **(v)** $300\pi\ m^2$.

Q18. **(i)** $y=\frac{27}{4x}-\frac{x}{4}$. **(ii)** $V=\frac{27x}{4}-\frac{x^3}{4}$. **(iii)** $\frac{27}{4}-\frac{3x^2}{4}$.

(iv) $x=3$ cm. **(v)** $13.5\ cm^3$.

Q19. **(i)** $y=6-3x$. **(ii)** $x^2\sqrt{3}+6(2x-x^2)$. **(iii)** $2x\sqrt{3}+12(1-x)$.

(iv) $\frac{12}{6-\sqrt{3}}$ m. **(v)** $\frac{12\sqrt{3}}{13-4\sqrt{3}}\ m^2$.

Q20. **(i)** $L=x+2y$. **(ii)** $L=x+\frac{2A}{x}$. **(iii)** $x=10\sqrt{2}$ units.

(iv) $y=5\sqrt{2}$ units. **(v)** $L=20\sqrt{2}$ units.

Q21. **(i)** $h=\frac{2000}{\pi r^2}$. **(ii)** $A=\frac{2000}{r}+\pi r^2+\frac{4000}{\pi r}$.

(iii) $\frac{dA}{dr}=-\frac{2000}{r^2}+2\pi r-\frac{4000}{\pi r^2}$. **(iv)** $r=10\left[\frac{(\pi+2)}{\pi^2}\right]^{1/3}$. **(v)** $\pi:(\pi+2)$.

Q22. **(i)** $\int_0^6 \frac{5}{6}\sqrt{36-x^2}\,dx$ **(ii)** $\int_{-6}^6 \frac{5}{6}\sqrt{36-x^2}\,dx$ **(iii)** 15π Sq.units.

(iv) 30π Sq.units. **(v)** 15π Sq.units.

Q23. **(i)** $(\sqrt{3},1),(-\sqrt{3},-1)$. **(ii)** $\frac{\pi}{3}$ Sq.units. **(iii)** π Sq.units.

(iv) 4π Sq.units.

Q24. **(i)** 2 Sq.units. **(ii)** 2 Sq.units. **(iii)** 2 Sq.units.

(iv) 2 Sq.units. **(v)** 2 Sq.units.

Q25. **(i)** $\frac{1}{3}$ Sq.units. **(ii)** $\int_0^1 (\sqrt{x}-x^2)\,dx$. **(iii)** $\frac{1}{3}$ Sq.units.

(iv) $\frac{1}{3}$ Sq.units. **(v)** $A_1=A_2=A_3$.

Q26. **(i)** $\frac{dP}{dt}=\frac{Pr}{100}$. **(ii)** $P=P_0\times e^{\frac{rt}{100}}$. **(iii)** $20\log_e 2=13.86$ years.

(iv) 6.93%. **(v)** ₹1648.

Q27. **(i)** Continuous and differentiable in $x\in(0,12)$. **(ii)** $m=-0.1$.

(iii) f is strictly increasing in (0, 6), f is strictly decreasing in (6, 12).

(iv) $x=6$ is the point of absolute maximum and the absolute maximum value of the function $=102.2$.

$x = 0, 12$ both are the points of absolute minimum and the absolute minimum value of the function $= 98.6$.

Q28. **(i)** $\pi y + 2x = 200$. **(ii)** $A = \frac{2}{\pi}(100x - x^2)$. **(iii)** $A = \frac{5000}{\pi}\ m^2$.

(iv) $S = \frac{2}{\pi}(100x - x^2) + \frac{1}{\pi} \times (100 - x)^2$, $\frac{dS}{dx} = -\frac{2x}{\pi}$, $x = 0$ m.

Q29. **(i)** $t = 4$ seconds. **(ii)** $\frac{32}{3}$ m.

Q30. **(i)** $-10x + 1250$. **(ii)** $x = 125$. **(iii)** ₹78155.

(iv) P(x) is strictly increasing in the interval $x \in (0, 125)$.

Q31. **(i)** $2r = x$. **(ii)** $\sqrt{3}r = h$. **(iii)** $\frac{\sqrt{3}}{6\pi}$ cm/s. **(iv)** $\frac{1}{2\pi}$ cm/s.

Q32. **(i)** $P(x) = -\frac{3x^2}{4} + 15x - 25$. **(ii)** $-\frac{3x}{2} + 15$. **(iii)** 10 units, ₹50.

(iv) 10 units, ₹50.

Q33. **(i)** $A = \frac{12}{5}x\sqrt{25 - x^2}$, $x \in (0, 5)$. **(ii)** $x = \frac{5}{\sqrt{2}}$.

(iii) Length : $5\sqrt{2}$ units, width : $3\sqrt{2}$ units.

(iv) Length : $5\sqrt{2}$ units, width : $3\sqrt{2}$ units.

Q34. **(i)** 1 metre/hour. **(ii)** 62.8 m^2/hour.

Q35. **(i)** $2x + y = 200$; $\therefore A(x) = 200x - 2x^2$ **(ii)** 5000 m^2.

Q36. **(i)** $V = \frac{\pi}{3} \times r^3$. **(ii)** $-\frac{1}{4\pi}$ cm/s. **(iii)** 2 cm^2/s. **(iv)** $-\frac{1}{4\pi}$ cm/s.

Q37. **(i)** For year 2000, value of t must be 0; $V(0) = -2$, which does not make any sense. So, the function V can not be used to estimate number of vehicles in the year 2000.

(ii) Show that $V'(t) > 0$.

Q38. **(i)** $C = 5000 \times x^2 + \frac{2500000000}{x^4}$. **(ii)** $\frac{dC}{dx} = 10000x - \frac{10000000000}{x^5}$.

(iii) $x = 10$ m.

(iv) C(x) is not an increasing function, when $x > 0$.

Q39. **(i)** Since h(t) is a polynomial function, so it is continuous everywhere when $t \geq 0$.

(ii) $t = \frac{13}{14}$ seconds.

Q40. **(i)** $V = \frac{\pi(75r - r^3)}{2}$. **(ii)** $\frac{dV}{dr} = \frac{\pi(75 - 3r^2)}{2}$.

(iii) $r = 5$ cm. **(iv)** The statement is false.

Q41. **(i)** $a = \frac{1}{27}$. **(ii)** $f''(1) = \frac{20}{27}$.

Q42. **(i)** $\frac{dy}{dx} = 4 - x$.

(ii) Rate of growth of the plant decreases for the first three days ; Height of the plant after 2 days is 6 cm.

Q43. **(i)** $\theta = \tan^{-1}\frac{5}{x}$ **(ii)** $-\frac{5}{x^2+25}$ rad/m **(iii)** $-\frac{4}{101}$ rad/s **(iv)** 15 m/s.

Q44. **(i)** $F = \frac{36}{5}$ **(ii)** $\frac{V}{250} - \frac{1}{4}$ **(iii)** $\frac{125}{2}$ km/h **(iv)** $\left(\frac{186}{5}\right) l$.

Q45. **(i)** $x = 450$ units **(ii)** ₹125.

Q46. **(i)** $A = 30 + 2x + \frac{72}{x}$ **(ii)** Length : 9 cm , breadth : 6 cm.

Q47. **(i)** $P = e^{kt+c}$ or, $P = \lambda e^{kt}$, where $\lambda = e^c$ **(ii)** $k = \log(2)$.

Q48. **(i)** (b) **(ii)** (a) **(iii)** (c) **(iv)** (a) **(v)** (d)

Q49. **(i)** (a) **(ii)** (c) **(iii)** (a) **(iv)** (b) **(v)** (a)

Q50. **(i)** (c) **(ii)** (a) **(iii)** (b) **(iv)** (d) **(v)** (c)

Q51. **(i)** (a) **(ii)** (b) **(iii)** (a) **(iv)** (c) **(v)** (a)

Q52. **(i)** (b) **(ii)** (c) **(iii)** (a) **(iv)** (b) **(v)** (c)

Q53. **(ii)** $y^2 - x^2 = Cx$

Q54. **(i)** $V = 2(2x^3 - 65x^2 + 500x)$ cm^3 **(ii)** $4(3x-50)(x-5)$

(iii) $x = 5$ cm **(iv)** $x = \frac{65}{6}$ is a point of inflection for V.

Q55. **(i)** $x = 400$ units

(ii) Darkest spot between the two light is at a distance of 400 units from a stronger lamp post i.e., at a distance of $600 - 400 = 200$ units from the weaker lamp post.

Q56. **(i)** $x^2 + y^2 = 100$; 8 m **(ii)** 0.45 m/s **(iii)** 1.05 m^2/sec

(iv) The top slides down faster when the ladder is farther from the wall.

Q57. **(i)** Strictly increasing in $x \in (0, 2)$; strictly decreasing in $x \in (2, \infty)$.

(ii) At $x = 2$ hours **(iii)** T(x) will always be greater than 25.

Unit IV - Vectors & 3 D Geometry

Q01. **(i)** $\hat{i} + 2\hat{j} + 3\hat{k}$. **(ii)** 2, –3, 4. **(iii)** 5.

(iv) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 3\hat{j} + 4\hat{k})$. **(v)** $\frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$.

Q02. **(i)** $-4\hat{i}$. **(ii)** $-\frac{5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$. **(iii)** $\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$.

(iv) $6\sqrt{3}\,\hat{k}$. **(v)** $3\sqrt{3}$ Sq. units.

Q03. **(i)** $\frac{\pi}{2}$. **(ii)** ΔABC is right angled and scalene triangle. **(iii)** $\vec{0}$.

(iv) $\frac{\sqrt{174}}{2}$ Sq.units. **(v)** $\sqrt{\frac{174}{35}}$.

Q04. **(i)** $-3\hat{i}+3\hat{j}$. **(ii)** $\hat{i}+\hat{j}+4\hat{k}$. **(iii)** $\frac{\hat{i}+7\hat{j}+10\hat{k}}{2}$.

(iv) $3\sqrt{2}$. **(v)** $3\sqrt{2}$.

Q05. **(i)** $\overrightarrow{OB}=5\hat{i}+3\hat{j}$. **(ii)** $\overrightarrow{OD}=8\hat{i}+8\hat{j}$. **(iii)** $\overrightarrow{BC}=-\hat{i}+4\hat{j}$

(iv) $6\sqrt{2}$. **(v)** $\sqrt{10}$.

Q06. **(i)** –1, 1, –3. **(ii)** $\frac{x-2}{-1}=\frac{y-3}{1}=\frac{z-5}{-3}$.

(iii) $-\frac{1}{\sqrt{11}},\frac{1}{\sqrt{11}},-\frac{3}{\sqrt{11}}$; $-\frac{1}{\sqrt{11}}\hat{i}+\frac{1}{\sqrt{11}}\hat{j}-\frac{3}{\sqrt{11}}\hat{k}$.

These values (d.c.'s) represent the unit vector in the direction of line AB, giving the precise angular orientation of the surveillance path in 3-D space.

(iv) AB is perpendicular to line (L). So, the warning should be triggered.

Q07. **(i)** 1, 2, –1; 2, 1, 1. **(ii)** (3, 3, 0).

(iii) S.D. = 0 units; the lines will intersect each other.

(iv) The lines will intersect each other. Motorcycles may collide at (1, 2, –1).

Q08. **(i)** 6 kN. **(ii)** Team A **(iii)** $\sqrt{2}$ kN **(iv)** $\frac{3\pi}{4}$.

Q09. **(i)** $\sqrt{113}$ units **(ii)** $\frac{5\hat{i}+2\hat{j}+4\hat{k}}{3\sqrt{5}}$ **(iii)** $\cos^{-1}\left(\frac{11}{45\sqrt{2}}\right)$ **(iv)** $\frac{11}{3\sqrt{5}}$.

Q10. **(i)** 4 **(ii)** $\cos^{-1}\frac{4}{\sqrt{42}}$.

Unit V - Linear Programming

Q01. **(i)** 12 **(ii)** $p=q$.

Q02. **(i)** $x+2y\geq 10,\ x+y\geq 6,\ 3x+y\geq 8,\ x\geq 0,\ y\geq 0$

(ii) Minimum value of $Z=112$ and it is obtained when $x=2,\ y=4$.

Unit VI - Probability

Q01. **(i)** 12%. **(ii)** 10%. **(iii)** 10.1%. **(iv)** $\frac{45}{101}$. **(v)** $\frac{81}{101}$.

Q02. **(i)** 1%. **(ii)** 12%. **(iii)** $\frac{34}{1000}$. **(iv)** $\frac{5}{34}$. **(v)** $\frac{5}{34}$.

Q03. **(i)** 0.9. **(ii)** 0.01 **(iii)** 0.999.

(iv) $\frac{10}{121}=0.0826\approx 0.083$ *or*, 8.3% **(v)** 0.01089.

Q04. **(i)** $\frac{3}{5}$. **(ii)** 1. **(iii)** $\frac{11}{15}$. **(iv)** 1. **(v)** $\frac{9}{11}$.

Q05.	**(i)**	$n=5$.	**(ii)**	$\frac{2}{5}$.	**(iii)**	$\frac{5}{12}$.	**(iv)**	1.	**(v)**	1.
Q06.	**(i)**	$\frac{200}{231}$.	**(ii)**	$\frac{31}{231}$.	**(iii)**	$\frac{231}{3000}$.	**(iv)**	$\frac{1}{231}$.	**(v)**	$\frac{230}{231}$.
Q07.	**(i)**	$\frac{2}{5}$.	**(ii)**	$\frac{1}{10}$.	**(iii)**	$\frac{13}{30}$.	**(iv)**	$\frac{1}{60}$.	**(v)**	$\frac{1}{5}$.
Q08.	**(i)**	$E_1 \cap E_3 = \{B_1B_2\}$.			**(ii)**	$\frac{1}{4}$.	**(iii)**	$\frac{1}{3}$.		
	(iv)	$\frac{1}{2}$.	**(v)**	1.						
Q09.	**(i)**	$\frac{12}{25}$ or, 48%.	**(ii)**	$\frac{2}{5}$ or, 40%.	**(iii)**	43%.	**(iv)**	88%.	**(v)**	$\frac{3}{5}$.
Q10.	**(i)**	0.999.	**(ii)**	0.001989.	**(iii)**	2000.	**(iv)**	$\frac{110}{221}$.		
	(v)	0.999.								
Q11.	**(i)**	$\frac{1}{2}$.	**(ii)**	$\frac{13}{150}$.	**(iii)**	$\frac{3}{26}$.	**(iv)**	$\frac{45}{52}$.	**(v)**	$\frac{1}{52}$.
Q12.	**(i)**	$\frac{9}{14}$.	**(ii)**	$\frac{5}{14}$.	**(iii)**	$\frac{1}{7}$.	**(iv)**	$\frac{5}{9}$.	**(v)**	$\frac{3}{5}$.
Q13.	**(i)**	$\frac{4}{7}$.	**(ii)**	$\frac{2}{7}$.	**(iii)**	$\frac{6}{7}$.	**(iv)**	$\frac{1}{7}$.	**(v)**	1.
Q14.	**(i)**	$\frac{21}{100}$.	**(ii)**	$\frac{5}{14}$.	**(iii)**	$\frac{2}{7}$.	**(iv)**	$\frac{1}{6}$.	**(v)**	$\frac{4}{21}$.
Q15.	**(i)**	$\frac{5}{6}$.	**(ii)**	$\frac{1}{6}$.	**(iii)**	$\frac{1}{3}$.	**(iv)**	$\frac{1}{2}$.	**(v)**	$\frac{2}{3}$.
Q16.	**(i)**	$\frac{2}{5}$.	**(ii)**	$\frac{4}{15}$.	**(iii)**	$\frac{1}{3}$.	**(iv)**	$\frac{3}{7}$.	**(v)**	$\frac{7}{10}$.
Q17.	**(i)**	$\frac{22}{425}$.	**(ii)**	$\frac{26}{425}$.	**(iii)**	$\frac{4}{17}$.	**(iv)**	$\frac{1}{26}$.	**(v)**	$\frac{1}{221}$.
Q18.	**(i)**	$\frac{5}{13}$.	**(ii)**	$\frac{4}{39}$.	**(iii)**	$\frac{20}{39}$.	**(iv)**	$\frac{390}{10000}$ or, 0.039.		
	(v)	$\frac{19}{39}$.								
Q19.	**(i)**	0.12 or, 12%.	**(ii)**	0.6.	Q20.	**(i)** 0.54.	**(ii)**	$\frac{7}{9}$.		
Q21.	**(i)**	$\frac{1}{4}$.	**(ii)**	$\frac{3}{4}$.	Q22.	**(i)** 0.30, 0.05.			**(ii)**	0.5.
Q23.	**(i)**	0.2.	**(ii)**	0.1.	**(iii)**	0.26.	**(iv)**	0.72, 0.02.		
Q24.	**(i)**	0.008.	**(ii)**	0.03.	**(iii)**	$\frac{17}{47}$.	**(iv)**	0.009, 0.047, 1.		

Q25. **(i)** 8.8%. **(ii)** $\frac{9}{44}$.

Q26. **(i)** $\frac{4}{10}, \frac{4}{10}, \frac{2}{10}$ **(ii)** 1.4. **(iii)** 49%. **(iv)** $\frac{16}{51}$.

Q27. **(i)** 0.35. **(ii)** 0.5075. **(iii)** $\frac{13}{29}$. **(iv)** $\frac{16}{29}$.

Q28. **(i)** 0.23. **(ii)** 0.04. **(iii)** $\frac{23}{36}$. **(iv)** 0.45.

Q29. **(i)** 0.17. **(ii)** 0.76. **(iii)** $\frac{1}{3}$. **(iv)** 0.39.

Q30. **(i)** 0.008. **(ii)** 0.047. **(iii)** $\frac{17}{47}$. **(iv)** 1.

Q31. **(i)** 0.38. **(ii)** $\frac{7}{19}$.

Q32. **(i)** $\frac{109}{300}$ **(ii)** $\frac{37}{109}$.

Q33. **(i)** 99.99999% **(ii)** 195% **(iii)** 99.9999995% **(iv)** $\frac{199999980}{199999999}$.

Q34. **(i)** $\frac{3}{5}$ **(ii)** $\frac{1}{3}$ **(iii)** $\frac{13}{30}$ **(iv)** $\frac{3}{20}$.

Q35. **(i)** 0.3 **(ii)** 0.4 **(iii)** 0.68 **(iv)** $\frac{14}{17}$.

Q36. **(i)** 0.59 **(ii)** $\frac{18}{59}$.

Q37. **(i)** $\frac{2}{5}$ **(ii)** $\frac{2}{15}$ **(iii)** $\frac{7}{15}$ **(iv)** $\frac{9}{15}$.

Q38. **(i)** $\frac{1}{5}$ **(ii)** $\frac{1}{15}$ **(iii)** $\frac{9}{30}$ **(iv)** $\frac{19}{30}$.

Q39. **(i)** $\frac{1}{2}$ **(ii)** $\frac{3}{10}$ **(iii)** $\frac{1}{2}$ **(iv)** $\frac{1}{10}$.

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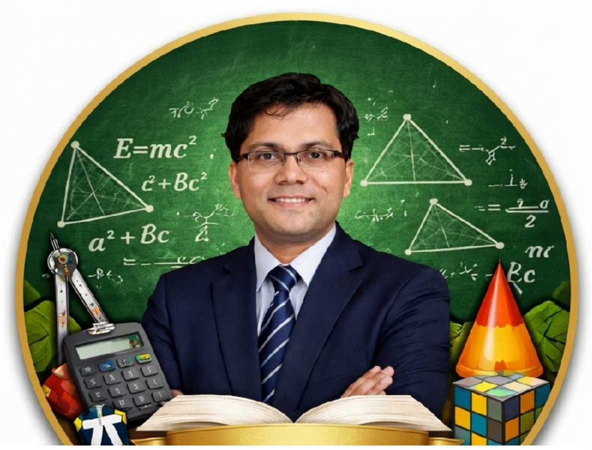

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ABOUT THE AUTHOR

O.P. GUPTA having taught math passionately over a decade, has devoted himself to this subject. Every book, study material or practice sheets, tests he has written, tries to teach serious math in a way that allows the students to learn math without being afraid. Undoubtedly his mathematics books are best sellers on amazon and Flipkart.

His resources have helped students and teachers for a long time across the country. He has contributed in CBSE Question Bank (issued in April 2021). Mr Gupta has been invited by many educational institutions for hosting sessions for the students of senior classes. Being qualified as an electronics & communications engineer, he has pursued his graduation later on with mathematics from University of Delhi due to his passion towards mathematics. He has been honored with the prestigious INDIRA AWARD by the Govt. of Delhi for excellence in education.

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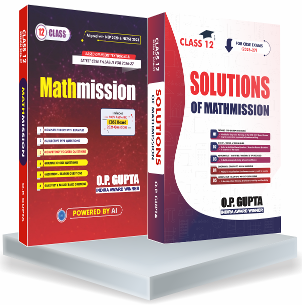

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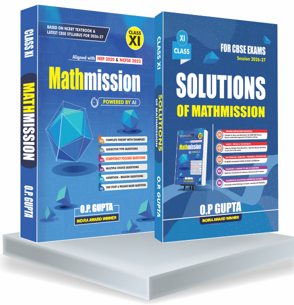

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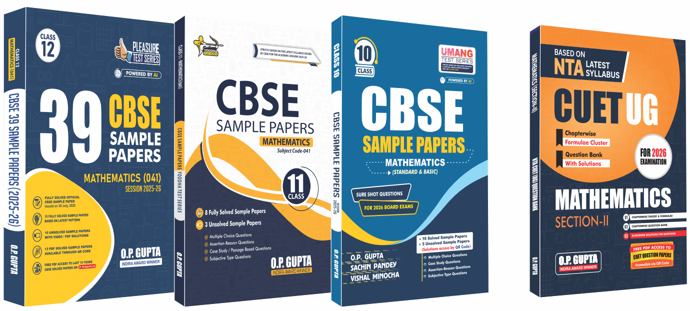

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